www.ijses.net

Constructing Mathematical Literacy Problems and Assessing Students' Solving Abilities

Eleni Nolka
Harokopio University of Athens, Greece

Chryssa Sofianopoulou
Harokopio University of Athens, Greece

To cite this article:

Nolka, E. & Sofianopoulou, C. (2025). Constructing mathematical literacy problems and assessing students' solving abilities. *International Journal of Studies in Education and Science (IJSES), 6*(2), 128-148. https://doi.org/10.46328/ijses.126

International Journal of Studies in Education and Science
2025, Vol. 6, No. 2, 128-148 https://doi.org/10.46328/ijses.126

Constructing Mathematical Literacy Problems and Assessing Students' Solving Abilities

Eleni Nolka, Chryssa Sofianopoulou

Article Info

Article History

Received:
24 August 2024
Accepted:
27 December 2024

Keywords

Mathematical literacy
Greece
Compulsory education
Coding

Abstract

The aim of the present study is to investigate students' abilities in mathematical literacy upon completing their compulsory education in Greece and to elucidate any challenges they may encounter. The research involved 650 students from various regions in Greece, selected based on the urbanization level of their respective school areas. These students had not undergone any training in mathematical literacy problem-solving or in expressing their thoughts in writing while undertaking such tasks. Mathematical literacy performance has been also assessed by the Programme for International Student Assessment (PISA) for the last two decades, in which the mean performance of Greek 15-year-old students has been consistently below the OECD average all these years. This discrepancy has been attributed, in part, to the inadequate alignment of the Greek mathematics curriculum with the PISA mathematics framework. To address this, the real-world problems constructed for this study were aligned with the Greek junior high school's mathematics curriculum. The findings indicate that students' mathematical literacy abilities upon completing compulsory education are limited, with the most significant challenges observed in the areas of mathematical reasoning and the written articulation of their thought processes.

Introduction

In the 21st century, the digitization of various aspects of our lives has prompted a reevaluation of the characteristics that individuals need to cultivate, with mathematics at the core. This shift emphasizes the importance of developing skills that enable active participation in the advancements of the modern world as thoughtful, creative, and engaged citizens (OECD, 2018). A crucial facet of this is the need for mathematical literacy skills to perceive, comprehend, and to some extent, navigate the quantitative components of society. Similar to how literacy in language empowers individuals, mathematical literacy, as asserted by Steen et al. (2007) also is empowering, which means that plays a pivotal role in fostering meaningful actions in people's daily lives. The significance of being mathematically literate, whether in reality or perception, carries global weight comparable to being literate in a more general sense (Howson, 2001). Consequently, schools are responsible for cultivating citizens who are both literate and mathematically literate.

The notion of mathematical literacy, first introduced in literature in 1944 (Niss & Jablonka, 2019), has posed a challenge for researchers in formulating a clear and universally accepted definition (Geiger et al., 2015; Jablonka, 2003; Skovsmose, 2007; Withnall, 1995). The difficulty in crafting a precise definition arises from the concept's focus not on the acquisition of mathematical knowledge itself but on its direct connection, as articulated by Jablonka (2003), to “the individual's own ability to use and apply this knowledge”. Furthermore, the role of mathematical literacy, according to Jablonka (2003), is to “teach people how to think, but not what to think”. Despite the challenge of achieving a common conceptual approach to the term, most researchers addressing mathematical literacy share a common objective: to foster in students, or in individuals at large, the ability to effectively apply their mathematical knowledge to meet challenges in everyday life and solve real-world problems (Gal & Tout, 2014; Goldenberg, 2014; Jablonka and Skilling, 2018; Madison & Steen, 2003; Niss & Jablonka, 2019; Steen et al., 2007).

The prominence of the concept of mathematical literacy has surged, particularly in the realm of international comparative research, largely propelled by the Programme for International Student Assessment (PISA). Organized triennially under the auspices of the OECD, PISA has played a pivotal role in rendering mathematical literacy a quantifiable metric. The initial and widely recognized attempt to establish a clear and shared definition dates back to the introduction of PISA's mathematical framework in 2000. Since then, the definition has been revised twice, acknowledging its consistency with earlier definitions of mathematical literacy while also addressing the evolving changes in our world.

Therefore, it is deemed highly crucial to nurture and enhance mathematical literacy skills, particularly within the school environment and specifically through mathematics classrooms. This equips students with the capacity to effectively navigate the challenges of contemporary society and their forthcoming lives. However, an important question arises: to what extent are current students in Greece, poised to become tomorrow's citizens, proficient in applying mathematics in their daily lives?

From the year 2000 until today, Greece has consistently taken part in PISA's programme. Throughout this period of engagement, the average score of Greek students in mathematical literacy has remained steady and consistently below the OECD average. This difference is statistically significant, amounting to approximately 40 score points, equivalent to a full school year (Nolka & Sofianopoulou, 2022). The consistent and modest performance average may find partial justification in the incongruity between the Greek mathematics curriculum and junior high school textbooks with the PISA mathematical literacy framework. Furthermore, the strong emphasis on content in these materials contributes to this trend (IEP, 2019; Nolka & Sofianopoulou, 2022; OECD, 2018). Despite this theoretical explanation, our pursuit of a deeper understanding of why Greek students face challenges in responding effectively to real-world problems led us to investigate how they approach such problems and uncover their misconceptions.

In this present study, our focus was on exploring students' abilities to tackle real-world problems. These problems were constructed within a framework aligned with both PISA's mathematical literacy framework and the Greek junior high school’s mathematics curriculum.

We aimed to address the following research questions:

(1) What is the mathematical literacy performance of students completing the compulsory education in Greece?
(2) Does the mathematical content of real-world problems impact students' performance in mathematical literacy?
(3) How do students approach mathematical reasoning and the solving of real-world math problems?
(4) What common errors and misconceptions do students exhibit when solving real-world math problems?

Literature Review

The European Commission, among its quantitative goals for 2020 outlined in the strategic framework "Education and Training 2020" (ET2020), included the objective of reducing the percentage of 15-year-old students demonstrating low performance in mathematics, to below 15% by the year 2020. Tracking the attainment of this goal relied on the outcomes of the 2018 PISA results, where European countries like Estonia, Denmark, Poland, and Finland successfully met this target. In contrast, Greece belonged to the group of countries where the percentage of students with low performance in mathematics exceeded 30% (European Commission, 2019). As the majority of EU countries did not attain this specific goal, it has been retained in the updated list of objectives for the next decade until 2030 (European Commission, n.d).

Over the past two decades, the PISA programme has conducted eight surveys, producing consistently updated and comparable results approximately every three years. These surveys contribute valuable data to educational research, offering insights into the mathematical literacy of 15-year-old students globally and, more specifically, in Greece. In addition to these PISA surveys, numerous studies in the literature involve secondary analyses of publicly available PISA data. Among these studies, a small number exclusively utilize the data from the representative sample of Greece (Cheema, 2018; Hiller et al., 2022; Karakolidis et al., 2016a, 2016b; Pitsia et al., 2017), either independently or in comparison with data from other participating countries (Kalaycioğlu, 2015; Lee, 2009; Martins & Veiga, 2010; Usta, 2016).

In parallel with the PISA surveys and their secondary meta-analyses, there are also a few studies that examine the performance and abilities in mathematical literacy of specific samples. These studies either construct their own instruments for measuring mathematical literacy performance (Dewantara et al., 2015; Ketonen & Hotulainen, 2019; Malassari et al., 2017; Oktiningrum et al. 2016; Pala et al., 2018; Sakonidis et al., 2017; Sari & Wijaya, 2017; Sidiropoulos, 2007) or employ real-world problems as assessment items from previous PISA surveys (Fointuna et al., 2020; Tariq et al., 2012). Sidiropoulos (2007), in his doctoral thesis, examined the mathematical literacy abilities of primary school graduates from Northern Greece. His research employed nine math word problems, revealing that students completing primary school faced challenges in successfully solving such problems (with an average performance of 3.1 points on a ten-point scale).

Focusing on students' performance in mathematical literacy problems and considering the mathematical content of the items, PISA 2012 revealed that, among OECD countries and in Greece separately, the most challenging subscale was “space and shape”. The easiest content subscale among OECD countries, with the highest overall average performance, was “quantity”, while Greece achieved the highest mean score in the “uncertainty and data” category.

In an extensive study conducted by Sakonidis et al. (2017), the mathematical literacy of minority students involved in the "Education of Muslim Children" program and majority junior high school students from all three grades (7^{th}, 8^{th} and 9^{th}) in the geographical region of Thrace was investigated. The participants responded to 19 questions distributed across the four branches of the mathematics curriculum in compulsory education: arithmetic, algebra, geometry, and stochastic mathematics. Overall, students' performance was average, with a statistically significant difference favoring majority students. The highest performance was observed in numeracy questions, followed by algebraic calculus, although with a significant difference from numerical calculus. Stochastic calculus followed, while the lowest performances were noted in geometric calculus questions. Aligning the four question categories based on the four calculations mentioned in Sakonidis et al.'s (2017) research with the four content categories of PISA problems, the “space and shape” category of PISA corresponded to the geometric calculus category. Notably, in Sakonidis et al.'s (2017) research, the lowest achievement rates were observed in this category, a finding consistent with the results of PISA 2012. Considering the category of arithmetic, if correlated with the “quantity” category, it is noteworthy that both studies—Sakonidis et al. (2017) and PISA 2012—indicated the highest success rates for the average performance of OECD countries as a whole. However, this did not align with the results for the Greek sample, as described above.

Examining the mean performance of students in PISA 2012 concerning the mathematical processes associated with each item, Greece exhibited the lowest mean score in “formulating” and the highest mean score in “interpreting” (OECD, 2014).

Theoretical Framework

The theoretical framework of this study incorporates several fundamental elements from PISA's mathematical framework while also aligning with the Greek junior high school’s mathematics curriculum. In recognition of the dynamic nature of the world, this survey adopts the most recent revised mathematics framework for PISA 2022, wherein mathematical literacy is defined as *“an individual’s capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens”* (OECD, 2018).

As per the aforementioned definition, we acknowledge the emphasis placed on both real-world problem-solving and mathematical reasoning to characterize someone as mathematically literate. A mathematically literate student must initially possess the ability to mathematically formulate situations, involving the recognition of opportunities to apply mathematics and the provision of a mathematical structure to a real-world problem. This restructuring necessitates mathematical reasoning. Secondly, mathematically literate students must be proficient in employing mathematical concepts, indicating their capacity to perform computations and manipulations. They should apply learned mathematical concepts, algorithms, and procedures to derive a mathematical solution to the formulated problem. Mathematical reasoning underlies this entire process. Thirdly, mathematically literate students should be adept at interpreting, applying, and evaluating mathematical results. This involves contemplating mathematical solutions, results, or conclusions and interpreting them within the context of real-world problems or situations. These processes are inherently intertwined with mathematical reasoning. Although mathematical reasoning and real-world problem-solving share common ground, mathematical reasoning extends beyond problem-solving and contributes to the cultivation of specific 21st-century skills. This encompasses the ability to make informed judgments about social issues addressable through mathematics and to assess the validity of quantitative and logical information. Moreover, two crucial aspects of mathematical reasoning include the inference from explicit hypotheses and statistical and probabilistic reasoning.

The analysis of the mathematical literacy framework also considers the mathematical content expected to be utilized in the assessment subjects. While the PISA framework primarily emphasizes students' abilities to apply mathematical knowledge to solve real-world problems, it presents an organizational structure for mathematical content characterized by diversity and depth, aiming to highlight crucial mathematical concepts. Simultaneously, it incorporates key aspects from various national mathematics curricula, although the Greek curriculum is not directly included. Therefore, in this study, we endeavored to align these categories more effectively with the Greek mathematics curriculum. This organizational structure encompasses four primary categories of mathematical content, ensuring breadth and depth related to traditional mathematical concepts without overly fine divisions that might compromise the richness of real-world situations presented in mathematical problems. These content categories are: (a) change and relationships, (b) space and shape, (c) quantity, and (d) uncertainty and data. The first category involves modeling change and relationships, utilizing appropriate functions and equations, as well as creating, interpreting, and translating symbolic and graphical representations of relationships. In the "space and shape" content category, a broad array of phenomena encountered in our visual and physical world is covered. The "quantity" content category incorporates the quantification of attributes in various contexts, understanding representations of those quantifications, and evaluating interpretations and arguments based on quantity. Lastly, the "uncertainty and data" content category involves recognizing the role of variation in processes, understanding the quantification of that variation, acknowledging uncertainty and error in measurement, and understanding chance. It also encompasses forming, interpreting, and evaluating conclusions in situations where uncertainty is critical.

Alignment with the Greek junior high school's curriculum in mathematics was achieved through relating the PISA mathematical content categories of the constructed items with the curriculum subject areas and objectives.

Method

Participants

The main study involved 650 students from across Greece who were either in the process of completing the 9th grade or commencing the 10th grade. The selection of schools for the sample was based on the degree of urbanization in the respective areas. Out of the participants, 386 (59.4%) attended schools situated in large urban areas (with more than 50,000 citizens), 162 (24.9%) attended schools in small urban areas (3,000<citizens≤50,000), and 102 (15.7%) attended schools in rural areas (≤3,000 citizens). Regarding gender distribution, 46.6% were boys, and 53.4% were girls. Initially, a trial-test survey was conducted on a sample of 50 ninth-grade students to evaluate the data collection process and the method of analysis (coding). This trial was also considered highly beneficial for refining the topics and questions for both tools used in the main research, ensuring they were clear and effective for data collection. Additionally, it helped determine the time required to implement the research in each class. This process significantly improved the initial research design by identifying potential problems and limitations, which were addressed before the main research was carried out. It is noteworthy that all students participating in the survey (both trial and main) had not undergone any training in mathematical literacy problem-solving or in expressing themselves in writing regarding their thought processes when dealing with real-world problems.

The Instrument

The tool used for data collection was a mathematical literacy test comprising 5 real-world problems divided into a total of 11 items. Specifically, 6 multiple-choice items and 5 open constructed-response items were created, and their scoring utilized analytical rubrics. The construction of the multiple-choice items aimed to include incorrect answer choices related to students' misconceptions (Wylie & Wiliam, 2006).

The open constructed-response items began with a disjunctive "Yes-No" question type, followed by the requirement for a written justification of their response to the disjunctive question. Coding of students' responses to these items provided additional insights into their misconceptions, common errors, and diverse approaches to solving real-world problems. The 11 items were categorized into three difficulty levels to accommodate the varied abilities of the participating students.

The assessment items were also classified into either mathematical reasoning or one of three mathematical processes associated with problem-solving. Specifically, there were 3 items invoking the process of formulating, 2 items for employing, 2 items for interpreting, and 4 items for mathematical reasoning. While mathematical reasoning could be observed within the items related to the three problem-solving processes, it contributed to only one domain. Additionally, an effort was made to achieve an approximate balance in the number of items between the two categories connecting the real world to the mathematical world (formulating and interpreting) and the other two categories where students worked on items with a mathematical form (employing and mathematical reasoning).

Regarding mathematical content knowledge, the constructed items were distributed across four content categories: 2 items belonged to the “change and relationships” category, 4 items to the “space and shape” category, 3 items to the “quantity” category, and 2 items to the “uncertainty and data” category.

Furthermore, the contexts of the constructed real-world math problems were developed to align with students' interests and situations relevant to their operations in the 21st century. Lastly, each item was matched with a designated thematic section of the Greek junior high school's curriculum, aligning with the corresponding objectives outlined in Table 1.

Table 1. Content of Mathematical Literacy Test

Item	1.Item Description 2.Link to the Greek curriculum (thematic section & curriculum objectives)	Content category	Process category	Degree of Difficulty	Question type
P1.1	1.The fundamental need for solving the problem involves establishing a mathematical model or formulating an algebraic relationship and calculation to arrive at the solution. 2. Ratio problems - 7th grade Organize the data of a proportion problem, solve proportion problems numerically.	Change & Relationships	Formulate	1	Multiple-choice
P1.2	1.Evaluating a ratio relationship by contrasting it with the numerical data of the problem to clarify the reasons behind an erroneous conclusion. 2. The function y=αx – 8th grade Organize the data of a ratio problem, solve ratio problems numerically. Determine the relationship between corresponding values of two proportional quantities.	Change & Relationship	Reasoning	2	Open-constructed
P2.1	Determining the quantity of individuals based on a specified criterion. 2. Ratio problems - 7th grade Organize the data of a proportion problem, solve proportion problems numerically.	Quantity	Formulate	1	Multiple-choice
P2.2	1.Assessing if everyone can be accommodated in the museum according to a specified criterion. 2. Areas of flat shapes & circle's measurement - 8th grade Comprehend the concept of surface area in relation to the chosen unit of measurement.	Space & Shape	Reasoning	3	Open-constructed

Item	1.Item Description 2.Link to the Greek curriculum (thematic section & curriculum objectives)	Content category	Process category	Degree of Difficulty	Question type
	Calculate the areas of squares, rectangles, and circular discs.				
P2.3	1.Determining the distance between two points utilizing the Pythagorean Theorem. 2. Pythagorean Theorem – 8th grade Understand the Pythagorean Theorem and apply it to solve associated problems.	Space & Shape	Formulate	2	Multiple-choice
P3.1	1.Computing and analyzing data related to mortality rates. 2. Percentages - 7th grade To solve percentage problems.	Uncertainty & Data	Interpret	2	Open-constructed
P4.1	1.Addressing a problem with fluctuating constraints. 2. Graphical representations – Pie charts – 8th grade To comprehend the usefulness of graphical representations and extract information from them.	Quantity	Reasoning	2	Multiple-choice
P4.2	1.Determining the updated available space by considering all the new numerical data. 2. Operations with decimal numbers – 7th grade To perform operations with decimal numbers.	Quantity	Employ	1	Open-constructed
P4.3	1.Create or generate a pie chart using the provided information. 2. Graphical representations – Pie charts – 8th grade To comprehend the usefulness of graphical representations and extract information from them. Create a pie chart based on the data presented in a table.	Uncertainty & Data	Interpret	2	Multiple-choice
P5.1	1.Identifying and employing the Pythagorean Theorem to solve the problem. 2. Pythagorean Theorem – 8th grade	Space &	Reasoning	3	Open-

Item	1.Item Description 2.Link to the Greek curriculum (thematic section & curriculum objectives)	Content category	Process category	Degree of Difficulty	Question type
	Understand the Pythagorean Theorem and apply it to solve associated problems.	shape			constructed
P5.2	1.Determining the coordinates of a point on the plane using specific data. 2. Level. Point. Addition and subtraction of line segments. Distance of points. Distance of a point from a straight line. Axis symmetry – 7th grade Perform addition and subtraction operations on line segments. Compute the distance of a point from a straight line. Identify instances where two points exhibit symmetry with respect to a line.	Space & Shape	Employ	2	Multiple-choice

The Problems

P1. Dimitris plans to walk to his friend's Antonis house. The "maps" application indicates that the distance is 1.6 km, and it would take him approximately 20 minutes to reach the destination.

P1.1) What is the approximate walking speed, in meters per minute, as calculated by the application?

A 125m/min B 80m/min C 8m/min D 12.5m/min

P1.2) Dimitris reached Antonis' house 4 minutes earlier than the time indicated by the application. Deciding to go together to the kiosk, they check the 'maps' app and find the nearest is 900m away. The app estimates it will take 11 minutes to walk there, but Dimitris asserts that they will arrive 4 minutes earlier if they walk at his pace. Is Dimitris correct? Yes or No? Explain your answer.

P2. Mike's class embarked on an educational visit to a museum with a total area of 200m². The group, comprising students and their accompanying teacher, totaled 20 individuals. At the museum entrance, there was a prominently displayed piece of paper with the following instructions written in large letters:

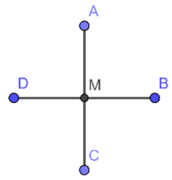
- Only 1 person per $15m^2$
- Minimum distance between people 1.5m

P2.1) Following the initial rule of a ratio of one person per 15m², what is the maximum capacity of people allowed to enter the museum simultaneously? A 15 B 13 C 14 D 10

P2.2) Mike asserts that by adhering to the second rule of maintaining a 1.5m distance from each other, all of them can enter the museum simultaneously. Do you agree with Mike? Yes or No? Provide your reasoning.

P2.3) The museum tour guide instructs them to form groups of five for an educational program while maintaining a 1.5m distance between individuals. Mike proposes a cross-shaped arrangement for them to stand on. If each student is positioned 1.5m away from the center M, how far apart will students A and B be?

A 2.12m B 1.5m C 3m D 4.5m



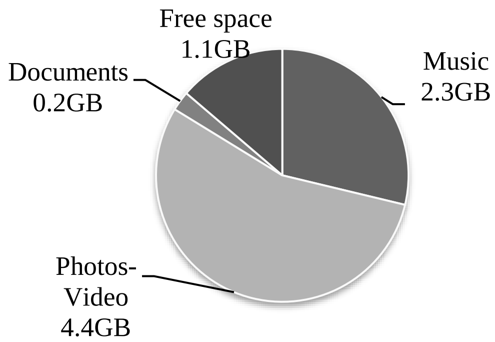
P3. In the previous year, the world faced a flu pandemic. The table below displays the number of outbreaks and deaths from the pandemic in five countries by the end of last December.

Table 2. Numbers of the Pandemic

Country	**Population**	**Outbreaks**	**Deaths**
A	10,700,000	2,530,000	36,624
B	10,720,000	1,540,000	21,479
C	11,560,000	2,290,000	28,518
D	67,220,000	14,600,000	150,000
E	83,240,000	7,550,000	114,000

P3.1) Examining the provided table, Vasia asserts that country E had a higher death rate than country A. Do you agree with Vasia? Yes or No? Explain your answer.

P4. Helen possesses an 8GB USB stick, where she stores photos-videos, documents, and music from her mobile phone. The circular diagram below illustrates the content stored on the stick.



P4.1) Helen intends to transfer 1.6 GB photos from her mobile phone to the USB stick. To make space for the new photos without deleting any photos-videos or documents, she opts to remove music files. Her preference is to delete up to three albums at most. The music albums currently on the stick are as follows:

ALBUM SIZE: Album 1: 525MB Album 2: 125MB Album 3: 475MB

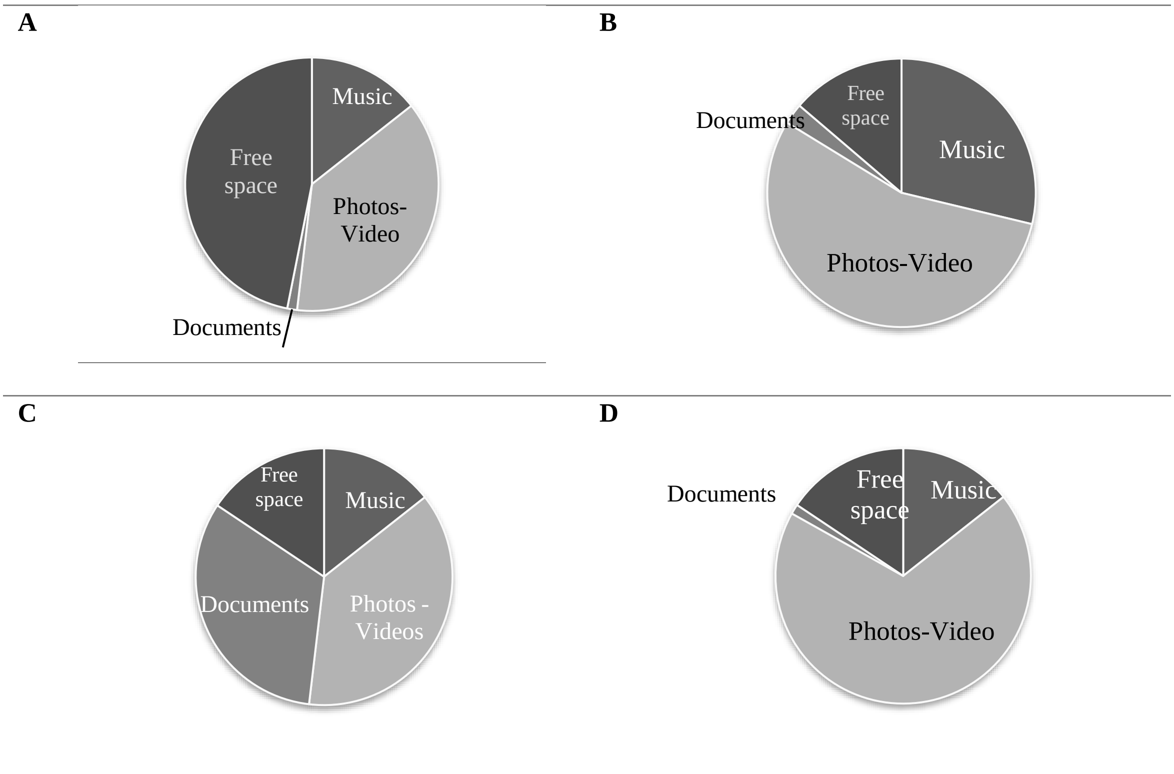
Album 4: 80MB Album 5: 100MB Album 6: 375MB Album 7: 55MB

By removing three music albums, will enough space be freed up to store all the new photos? (1GB ≈ 1000MB). Yes or No? Explain your answer.

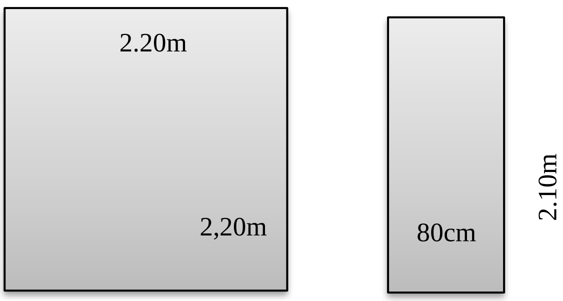
P4.2) Helen's sister gifted her a new 16GB memory stick to avoid the need for deleting her music albums. What will be the new available space on the new stick after transferring everything from the previous stick, including the new photos from her mobile phone?

A 6GB B 7.5GB C 2.3GB D 9.6GB

P4.3) Which of the four pie charts below illustrates the distribution of content on Helen's new 16GB USB stick, encompassing both the new photos and everything from her previous stick?



P5. In the art class, the students crafted a substantial square structure using solid materials. Once finished, they decide to display it on the wall of their classroom. However, to transport it from the art room to their classroom, they need to navigate through two doors. Both doors share the same dimensions, measuring 2.10m in height and 80cm in width. The square construction itself has a side length of 2.20m.

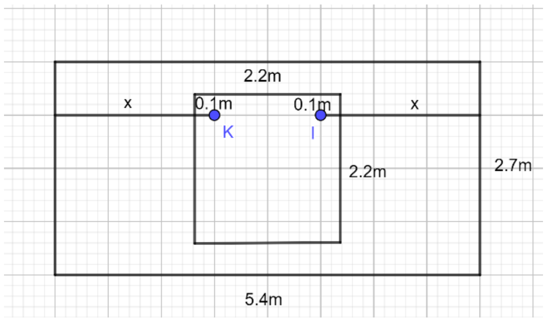


P5.1) The majority of students, upon observing the construction, claim that it will not fit through the doors. However, Marina contends that it will fit. Is Marina correct? Yes or No? Explain your answer.

P5.2) After the students have devised a way to move their construction into the classroom, they aim to hang it on the wall. The wall, measuring 5.4m in length and 2.7m in height, is where they intend to position it at the center. In their planning, the students are determining where to place the nails. The stud calculations suggest positioning them at points I and K, as illustrated in the figure below. What is the distance "x" (from the left edge of the wall to K or from the right edge of the wall to I) that the students need to calculate? Select a numerical value for "x."

A x=1.7m B x=1.6m

C x=1.1m D x=2.7m



Results

Table 3 shows that the average number of the mathematical test items that were solved correctly by the students was 4.27 out of the eleven in total test items, with a standard deviation of 2.63. The average number of correctly solved multiple-choice items was 3.16 (SD=1.74) out of the six multiple-choice items in total, while among the five open-constructed response items the average number of correctly solved items was only 1.1 (SD=1.19).

Table 3. Number of Correct Items per Student

Items	**M**	**SD**	**SE**	**Q1**	**Q2**	**Q3**	**Min**	**Max**
All items	4.27	2.63	.10	2	4	6	0	11
Multiple-choice	3.16	1.74	.10	2	3	5	0	6
Open-response	1.1	1.19	.05	0	1	2	0	5

The assessment of students' mathematical literacy performance involved coding their responses and categorizing them as correct, partially correct, incorrect, or no-answer. Additionally, the calculation of weighting coefficients for each item was conducted, taking into account the graded difficulty of individual items. The coding of responses to open-constructed items is detailed in Table 4.

Table 4. Correct and Partial Correct Answers - Coding

Answer		**Code**	**Description**	**Frequency (%)**
P1.2	Correct	2.1	"No" and provide a valid explanation supported by relevant numerical values.	136(20.9%)
		2.2	"No" and offer a valid rationale without relying on numerical data. In the justification, they demonstrate an understanding of all three critical factors—speed, distance, and the four-minute time difference—that must be considered for a correct response.	29(4.5%)
			"No" and they provide a justification that is partially acceptable without specifying numerical values and	78(12%)

Answer		Code	Description	Frequency (%)
	Partial correct	1.1	without explicitly addressing all three conditions. They include one or two factors, such as speed or distance, in their explanation, but omit the reference to the four-minute time difference.	
P2.2	Correct	2.1	"Yes" and give an acceptable justification with numerical values or an acceptable analysis of the data.	48(7.4%)
	Partial correct	1.1	"Yes" or "No" and provide a partial explanation or one with minor numerical errors.	23(3.5%)
P.3.1	Correct	2.1	"No" and provide acceptable numerical values or conduct a satisfactory numerical analysis for both countries.	107(16.5%)
		2.2	"No" and provide a suitable and acceptable analysis of the data without relying on numerical values	113(17.4%)
	Partial Correct	1.1	"Yes" while simultaneously supporting a "No" answer with partially correct explanation.	49(7.5%)
		1.2	"No" and provide an explanation, with or without numerical values, for either one of the two countries.	20(3.1%)
		1.3	"No" or "Yes" and compare the deaths with the outbreaks rather than considering the population.	34(5.2%)
P4.1	Correct	2.1	"Yes" and they leverage the presence of 1.1GB of free space on the USB stick. Furthermore, they identify specific music albums for deletion, and the cumulative size surpasses the required 1.6GB.	172(26.5%)
		2.2	"Yes", utilize the available 1.1GB of free space, asserting that it will accommodate the content without explicitly specifying three albums or fewer.	36(5.5%)
	Partial correct	1.1	"No" and they claim that there are no three albums whose combined size reaches or exceeds 1.6GB, without considering the existing 1.1GB of free space.	162(24.9%)
		1.2	"No" but their response provides a justification for selecting "Yes."	12(1.8%)
P5.1	Correct	2.1	"Yes" and the justification involves numerical data, including the calculation of the doors' diagonal.	75(11.5%)
	Partial correct	1.1	"Yes" and reason that it should be entered diagonally without providing numerical documentation or proof.	147(22.6%)
		1.2	"Yes" or "No" with minor numerical errors.	7(1.1%)

Upon evaluating the mathematical tests of all study participants (N=650), it was determined that the average performance in mathematical literacy for students was 16.38 (SD=10.02), with a maximum possible score of 42 units. Half of the surveyed students attained scores up to 14 points. A quarter of the sample achieved a score equal to or greater than 24 points, while another 25% scored less than or equal to 8 points (refer to Table 5).

Table 5. Students' Average Performance in Mathematical Test

M	SD	SE	Q1	Q2	Q3	Min	Max
16.38	10.02	.39	8	14	24	0	42

Table 6 shows that, out of the eleven items, students achieved correct answer percentages above 50% on four of them. The highest percentage of correct answers, surpassing 73%, was observed in the first item P1.1, which was a multiple-choice question falling under the content category "change and relationships" and the process category "formulating." Another notable performance was seen in the multiple-choice item P2.1, with nearly 66% of students providing correct answers. This item also belonged to the "formulating" process but was categorized under "quantity." However, the open-response items P5.1 and P2.2 presented challenges, with less than 12% and slightly over 7% of students, respectively, answering them fully correctly. Both items belonged to the content category "space and shape" and invoked "mathematical reasoning."

Among the multiple-choice items and open-response items, the highest rates of correct answers were observed in the former. Notably, the multiple-choice item P5.2 ("space and shape" & "employing") recorded the lowest percentage of correct answers, approximately 30%. It is noteworthy that this item was the last one in the sequence of the research tool. Overall, students displayed greater difficulty in handling and offering a well-documented response to open-response items compared to multiple-choice items.

Referring to Table 6, it is evident that certain items exhibited noteworthy proportions of partially accepted answers. Specifically, in item P5.1, which garnered the second-lowest percentage of overall acceptable responses, partially correct answers (≈ 24%) were twice as prevalent as completely correct ones (≈12%). Similarly, in item P5.2, the percentage of partially correct answers (approximately 29%) closely mirrored that of fully correct responses (≈ 30%). Significant proportions of partially correct answers, in comparison to fully correct ones, were also notable in items P4.1 and P3.1. Moreover, items P4.1 and P5.2 displayed similar percentages across the three categories of answers: correct, partially correct, and incorrect.

Table 6. Students' Performance in Items

Item	Correct answer	Partial correct answer	Incorrect answer	No answer
P1.1	476(73.2%)	68(10.5%)	79(12.2%)	27(4.2%)
P2.1	427(65.7%)	58(8.9%)	135(20.8%)	30(4.6%)
P4.2	352(54.2%)	-	225(34.6%)	73(11.2%)
P4.3	346(53.2%)	-	191(29.4%)	113(17.4%)
P2.3	257(39.5%)	46(7.1%)	291(44.8%)	56(8.6%)
P3.1	221(34%)	104(16%)	264(40.6%)	61(9.4%)

Item	Correct answer	Partial correct answer	Incorrect answer	No answer
P4.1	208(32%)	174(26.8%)	228(35.1%)	40(6.2%)
P5.2	197(30.3%)	185(28.5%)	163(25.1%)	105(16.2%)
P1.2	165(25.4%)	78(12%)	382(58.8%)	25(3.8%)
P5.1	76(11.7%)	153(23.5%)	348(53.5%)	73(11.2%)
P2.2	48(7.4%)	23(3.5%)	501(77.1%)	78(12%)

Figure 1 scrutinizes students' mathematical literacy performance in relation to the mathematical content categories of the items. Notably, the "space and shape" category exhibited a significant and adverse contrast when compared to the other three content categories as a whole. Specifically, among the items falling under the content categories of "change and relationships," "quantity," and "uncertainty and data," which recorded the highest percentages of correct answers, variations and alternations in percentages were evident based on the type of analysis conducted (considering all items or categorizing them into open-response or multiple-choice, or based on the degree of difficulty). Conversely, for items garnering the highest percentages of fully accepted responses, there was no uniformity in results, except for the "space and shape" category, which consistently exhibited the lowest percentages of fully accepted responses.

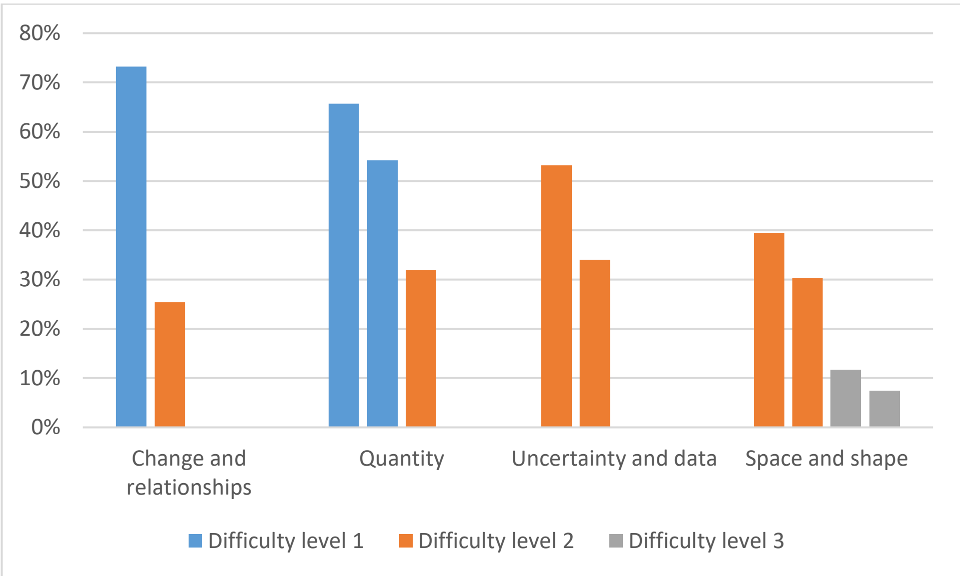


Figure 1. Correct Answers by Mathematical Content Category

Furthermore, the items in the "space and shape" category exhibited notable distinctions in terms of their percentages of partially correct answers compared to the corresponding figures for items in the other three categories. Specifically, in three out of the four items within this category, the percentages of partially accepted answers were significantly higher and noteworthy when compared to the corresponding percentages of fully accepted answers.

In summary, it can be inferred that students faced a higher level of difficulty in handling problems or items belonging to the "space and shape" mathematical content category in comparison to items from the other three categories: "change and relationships," "quantity," and "uncertainty and data". The "space and shape" category can be characterized as the least robust among the four content areas.

In Figure 2, students' mathematical performance in examined in relation to the process categories and the mathematical reasoning. If the degree of difficulty and the type of questions of the items were not taken into account, the two largest percentages of accepted answers were concentrated in two items which activated "formulating" as a basic mathematical process. However, taking into account together with the mathematical process and the type of item (open-response or multiple choice) and the degree of difficulty, there is no uniformity in students' performance in the items that mainly invoked the three problem solving categories, formulating, employing and interpreting. However, in the items that invoked the mathematical reasoning, which is referred to as the most complex process since it presupposes or includes all the other three processes, formulate, employ, interpret, the lowest percentages of acceptable answers were noted by the students as a whole compared to the corresponding percentages in the items that recognized the other three as main processes. Even when partially correct answers were taken into account, the percentages of partially accepted answers in the open-constructed response items that were invoked mathematical reasoning, show remarkable percentages and quite high compared to the corresponding percentages of accepted answers.

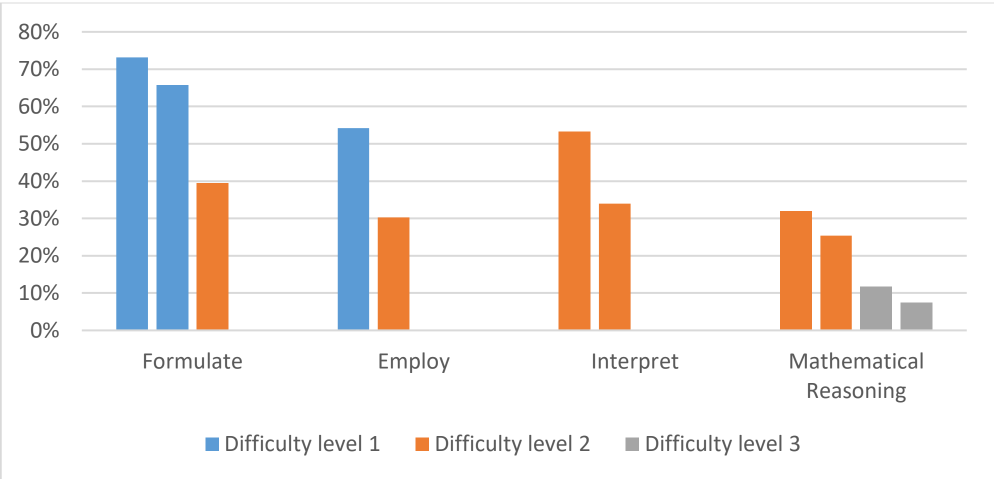


Figure 2. Correct Answers by Item & Mathematical Procedure

In Figure 2, students' mathematical performance is assessed in relation to the process categories and mathematical reasoning. Without considering the degree of difficulty and the type of questions in the items, the two highest percentages of accepted answers were concentrated in items that activated "formulating" as a fundamental mathematical process. However, when considering the mathematical process alongside the type of item (open-response or multiple choice) and the degree of difficulty, there is no consistency in students' performance in items primarily invoking the three problem-solving categories: formulating, employing, and interpreting. In contrast, in items invoking mathematical reasoning, recognized as the most complex process encompassing all three processes (formulating, employing, interpreting), students, as a whole, achieved the lowest percentages of accepted answers compared to items emphasizing the other three processes as main components.

Even when factoring in partially correct answers, the open-constructed response items that invoked mathematical reasoning exhibited notable and relatively high percentages of partially accepted answers, surpassing the corresponding percentages of fully accepted answers.

After analyzing students' incorrect responses, their errors were categorized into three distinct groups (see Table 7). It is evident that all three coded categories accumulated noteworthy percentages of errors. The predominant share of incorrect answers is attributed to undocumented responses, underscoring the challenge students face in articulating and justifying their choices in written form. Subsequent significant percentages of errors highlight students' struggles with the comprehensive process of mathematization, encompassing stages such as formulating, employing, interpreting, evaluating, and mathematical reasoning. Ultimately, a significant number of students encounter challenges in recognizing or interpreting a situation presented within a real-world context. Ultimately, a significant number of students encounter challenges in recognizing or interpreting a situation presented within a real-world context. Instead, they provide a straightforward and incorrect conclusion, merely repeating numerical data from the problem without offering any elaboration, formulation, or application of mathematical reasoning in their answers.

Table 7. Students' Errors

Type of error	**P1.2 (%)**	**P2.2**	**P3.1**	**P4.1**	**P5.1**
Inaccurate mathematical reasoning or the absence of a proper mathematical argument	147(22.6%)	212(32.6%)	39(6%)	57(8.8%)	137(21.1%)
Employ or replicate numerical data from the problem in the response without providing any formulation or reasoning.	103(15.8%)	103(15.8%)	133(20.5%)	60(9.2%)	61(9.4%)
Choose either yes or no without providing any written justification.	132(20.3%)	186(28.6%)	94(14.5%)	111(17.1%)	150(23.1%)

Discussion

Regarding students' performance in mathematical literacy, the analysis of descriptive results from the research tool, the mathematical test, indicates a lack of satisfactory mathematical literacy skills among students. Specifically, the sample's average performance was only 16.38 points, with a maximum score of 42 points and a median value of 14. Across the eleven items assessed in the math test, students, on average, provided correct answers to 4.27 items. When distinguishing between multiple-choice and open-response items, students, on average, correctly answered 3.16 out of the six multiple-choice items and fully addressed only 1.1 out of the five open-response constructed items. These findings align to some extent with the outcomes of the international PISA survey, reflecting a low level of mathematical literacy among Greek students (OECD 2003, 2004, 2007, 2010, 2014, 2016, 2019, 2023). Additionally, similar patterns are observed in other related studies, such as Sidiropoulos (2007) with a sample of primary school students.

Concerning the mathematical content of the items under examination, students faced a higher level of difficulty in managing items falling within the mathematical content category of "space and shape" compared to items from the other three categories, namely "change and relationships," "quantity," and "uncertainty and data." This observation aligns with the overall results of the PISA 2012 survey for OECD countries, including a representative sample from Greece (OECD, 2014). Additionally, drawing parallels between the item categories in the research by Sakonidis et al. (2017) and aligning the "geometric calculus topics" category with the "space and shape" category in the current study, both surveys concurred on the greater difficulty students faced in handling issues related to this category compared to subjects from the other three categories.

In an examination of students' responses to items based on their invoked mathematical processes, it was revealed that items pinpointing mathematical reasoning posed greater difficulty for students. The concept of reasoning, recognized as a pivotal aspect of mathematical literacy, emerges in the assessment framework of the latest PISA 2022 (OECD, 2018). Consequently, this represents a novel concept warranting exploration, as existing literature lacks published research for comparative purposes, underscoring the significance of this particular discovery. Among the three problem-solving processes, the two items garnering the highest percentages of acceptable responses across all mathematical test items focused on the “formulating” process. Notably, these items did not lend themselves to generalization, as they were both multiple-choice and of difficulty level 1. In PISA 2012, items assessing the “formulating” process garnered the lowest average scores for both the Greek sample and OECD countries. In the present research, an item involving the “interpretating” process recorded the highest percentage of acceptable answers among difficulty level 2 items. While this observation might hint at the importance of the interpretating process, caution is advised against broad generalization.

Conclusion

Upon analyzing all the results, it becomes evident that a significant majority of students upon completing their compulsory education encountered challenges in handling real-world problems. They struggled to identify and effectively implement the stages of mathematical modeling, encompassing formulating, employing, interpreting, and evaluating. Notably, mathematical reasoning, integral to these processes, posed a considerable difficulty for the students. In essence, the findings demonstrate that students faced challenges in tackling problems that emphasize mathematical literacy skills. Consequently, it can be inferred that they did not possess satisfactory levels of mathematical literacy skills.

Based on the findings of this research, it is crucial to cultivate mathematical literacy in mathematics classes, as these skills are essential for everyone to participate fully and equitably in our ever-evolving 21st-century society. Enhancing mathematical reasoning and the process of mathematization, along with emphasizing the verbal expression of students' thought processes during teaching, can play a significant role in this development. Integrating more real-world problems into math lessons and systematically analyzing students' responses to these challenges are also valuable strategies for educators. By implementing these approaches, we can work toward nurturing a greater number of mathematically literate students and future citizens.

References

Breakspear, S. (2012). The Policy Impact of PISA: An Exploration of the Normative Effects of International Benchmarking in *School System Performance. OECD Education Working Papers*, No. 71. Paris: OECD. https://doi.org/10.1787/5k9fdfqffr28-en

Cheema, J. R., (2018). Effect of math-specific self-efficacy on math literacy: Evidence from a Greek survey. *Research in Education*, 102(1), 13–36. https://doi.org/10.1177/0034523717741914.

Dewantara, A. H., Zulkardi, Z., & Darmawijoyo, D. (2015). Assessing Seventh Graders' Mathematical Literacy in Solving PISA-like Tasks. *Journal of Mathematics Education*, *6*, 117–128.

European Commission, (n.d.) *European education Area. Quality education and training for all. Strategic framework.* https://education.ec.europa.eu/about-eea/strategic-framework

Fointuna, D. W., Kaluge, A. H., & Fernandez, A. J. (2020). An analysis of mathematical literacy of state junior high school students in Kupang. In *Journal of Physics: Conference Series* (Vol. 1422, No. 1, p. 012025). IOP Publishing.

Gal, I., & Tout, D. (2014). Comparison of PIAAC and PISA Frameworks for Numeracy and Mathematical Literacy. *OECD Education Working Papers*, *102*(102), 58. https://doi.org/10.1787/5jz3wl63cs6f-en

Geiger, V., Forgasz, H., & Goos, M. (2015). A critical orientation to numeracy across the curriculum. *ZDM–Mathematics Education*, 47(4), 611-624.

Goldenberg, E. P. (2014). "Mathematical literacy": an inadequate metaphor. In M. N. Fried, & T. Dreyfus (Eds.), *Mathematics & mathematics education: searching for common ground* (pp. 139-156). New York: Springer.

Hiller, S. E., Kitsantas, A., Cheema, J. E., & Poulou, M. (2022). Mathematics anxiety and self-efficacy as predictors of mathematics literacy. *International Journal of Mathematical Education in Science and Technology*, *53*(8), 2133-2151. DOI: 10.1080/0020739X.2020.1868589

IEP. (2019). *Greek Mathematics Education and PISA assessment*, Athens: Institute of Educational Policy. https://iep.edu.gr/images/IEP/EPISTIMONIKI_YPIRESIA/Epist_Grafeia/Graf_Ereynas_B/PISA/2016_Apologismos_IEP.pdf

Jablonka, E. (2003). Mathematical literacy. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (Vol. 1, pp. 75–102). Dordrecht: Kluwer Academic Publishers.

Jablonka, E. & Skilling, K. (2018). Numeracy, mathematical literacy and mathematics. In Maguire, M., Gibbons, S., Glackin, M., Pepper, D. & Skilling, K. (Eds.), *Becoming a Teacher: Issues in secondary education* (5th Edition). Maidenhead: McGraw Hill.

Kalaycioğlu, D. B. (2015). The influence of socioeconomic status, self-efficacy and anxiety on mathematics achievement in England, Greece, Hong Kong, the Netherlands, Turkey, and the USA. *Educational Sciences: Theory & Practice*, 15(5), 1391–1401. https://doi.org/10.12738/estp.2015.5.2731

Karakolidis, A., Pitsia, V., & Emvalotis, A. (2016a). Examining students' achievement in mathematics: A multilevel analysis of the Programme for International Student Assessment (PISA) 2012 data for Greece. *International Journal of Educational Research*, *79*, 106-115.

Karakolidis, A., Pitsia, V., & Emvalotis, A. (2016b). Mathematics low achievement in Greece: A multilevel analysis of the Programme for International Student Assessment (PISA) 2012 data. *Themes in Science and Technology Education*, *9*(1), 3-24.

Ketonen, E. E., & Hotulainen, R. (2019). Development of low-stakes mathematics and literacy test scores during lower secondary school–A multilevel pattern-centered analysis of student and classroom differences. *Contemporary Educational Psychology, 59*, 101793.

Lee, J. (2009). Universals and specifics of mathematics self-concept, mathematics self-efficacy and mathematics anxiety across 41 PISA 2003 participating countries. *Learning and Individual Differences*, 19(3), 355–365. https://doi.org/10.1016/j.lindif.2008.10.009

Madison, B. L., & Steen, L. A. (Eds.). (2003). *Quantitative literacy: Why numeracy matters for schools and colleges*. National Council on Education and the Disciplines. Princeton.

Malasari, P. N., Herman, T., & Jupri, A. (2017, September). The construction of mathematical literacy problems for geometry. In *Journal of Physics: Conference Series* (Vol. 895, No. 1, p. 012071). IOP Publishing.

Martins, L., & Veiga, P. (2010). Do inequalities in parents' education play an important role in PISA students' mathematics achievement test score disparities?. *Economics of Education Review*, *29*(6), 1016-1033.

Nolka, E., & Sofianopoulou, C. (2022). Greek and Portuguese Mathematics education and Performance, through the prism of PISA. In M. Carmo (Ed*.) Education Applications & Developments VII, Chapter 1* (pp. 3–12). Lisbon, InScience Press. https://doi.org/10.36315/2022ead01

Niss, M., & Jablonka, E. (2019). Mathematical Literacy. In: Lerman, S. (eds) Encyclopedia of Mathematics Education. Springer, Cham. https://doi.org/10.1007/978-3-319-77487-9_100-5

OECD. (2003). The PISA 2003 Assessment Framework*: Mathematics, Reading, Science and Problem Soloving Knowledge and Skills.* OECD. https://doi.org/10.1787/9789264101739-en

OECD. (2004). *Learning for Tomorrow's World: First Results from PISA 2003*, PISA, OECD Publishing, Paris. https://doi.org/10.1787/9789264006416-en.

OECD. (2007). PISA 2006: *Science Competencies for Tomorrow's World: Volume 1:* Analysis, PISA, OECD Publishing, Paris. https://doi.org/10.1787/9789264040014-en.

OECD. (2010). *PISA 2009 Results: What Students Know and Can Do: Student Performance in Reading, Mathematics and Science (Volume I)*. Paris: OECD Publishing.

OECD. (2014). *PISA 2012 Results: What Students Know and Can Do – Student Performance in Mathematics, Reading and Science (Volume I, Revised edition, February 2014*), PISA, OECD Publishing. *http://dx.doi.org/10.1787/9789264201118-en*

OECD. (2016). *PISA 2015 Results (Volume I): Excellence and Equity in Education*. PISA, OECD Publishing, Paris. https://doi.org/10.1787/9789264266490-en.

OECD. (2018). *Pisa 2021 Mathematics Framework (First Draft)*. https://www.upc.smm.lt/naujienos/smm/penkiolikmeciu-matematinis-rastingumas/GB-2018-4-PISA-2021-Mathematics-Framework-First-Draft.pdf

OECD. (2019). *PISA 2018 Results (Volume I): What Students Know and Can Do*. PISA, OECD Publishing, Paris. https://doi.org/10.1787/5f07c754-en

OECD. (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education.* PISA, OECD Publishing, Paris, https://doi.org/10.1787/53f23881-en.

Oktiningrum, W., Zulkardi, Z., & Hartono, Y. (2016). Developing PISA-like mathematics task with Indonesia natural and cultural heritage as context to assess students mathematical literacy. *Journal on Mathematics Education*, *7*(1), 1-8.

Pala, R. H., Herman, T., & Prabawanto, S. (2019). Students' error on mathematical literacy problems. In *Journal of Physics: Conference Series* (Vol. 1157, No. 2, p. 022125). IOP Publishing.

Pitsia, V., Biggart, A., & Karakolidis, A. (2017). The role of students' self-beliefs, motivation and attitudes in predicting mathematics achievement: A multilevel analysis of the Programme for International Student Assessment data. *Learning and Individual Differences*, *55*, 163-173.

Sari, R. H. N., & Wijaya, A. (2017). Mathematical literacy of senior high school students in Yogyakarta. *Jurnal Riset Pendidikan Matematika*, *4*(1), 100-107.

Sakonidis, C., Klothou, A., Fakoudis, E., Margaritis, C., Antampoufis, N., Dramalidis, A., & Nizam, A. (2017). Aspects of mathematical literacy: The case of Thracian minority students. *Proceedings of the 7th Panhellenic Conference of the En.E.Di.M.* Athens.

Sidiropoulos, D. A. (2007). *Mathematical literacy of primary school graduates and the in-school and out-of-school factors that influence it* [Unpublished Doctoral dissertation]. Aristotle University of Thessaloniki.

Skovsmose, O. (2007). Mathematical literacy and globalisation. *Internationalisation and globalisation in mathematics and science education*, 3-18.

Steen, L.A., Turner R., & Burkhardt H. (2007). Developing Mathematical Literacy. In: Blum W., Galbraith P.L., Henn HW., Niss M. (eds) *Modelling and Applications in Mathematics Education.* New ICMI Study Series, vol 10. Springer, Boston, MA. https://doi.org/10.1007/978-0-387-29822-1_30

Tariq, V. N., Qualter, P., Roberts, S., Appleby, Y., & Barnes, L. (2013). Mathematical literacy in undergraduates: role of gender, emotional intelligence and emotional self-efficacy, *International Journal of Mathematical Education in Science and Technology*, 44:8, 1143-1159, DOI: 10.1080/0020739X.2013.770087

Withnall, A. (1995). Towards a definition of numeracy. In *Adults learning maths–A research forum ALM-1: Proceedings of the inaugural conference of adults learning maths–A research forum* (pp. 11-17).

Wylie, E. C., & Wiliam, D. (2006). *Analyzing diagnostic items: What makes a student response interpretable?* In Annual Meeting of the National Council on Measurement in Education (NCME). Chicago, IL.

Usta, H. G. (2016). Analysis of student and school level variable related to mathematics self-efficacy level based on PISA 2012 results for China-Shanghai, Turkey, and Greece. *Educational Science: Theory & Practice*, *16*(4), 1297–1323. https://doi.org/10.12738/estp.2016.4.0283

Author Information

Eleni Nolka
https://orcid.org/0009-0008-1970-8427
Harokopio University of Athens
9, Omirou Str., 17778, Tavros, Athens
Greece
Contact e-mail: *eleninol@yahoo.gr*

Chryssa Sofianopoulou
https://orcid.org/0000-0001-8575-0908
Harokopio University of Athens
9, Omirou Str., 17778, Tavros, Athens
Greece