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# University Students' Preferences of Representation Types in Learning Calculus

**Nigusse Arefaine Gebru, Kassa Michael Wyesus, Shimelis Assefa Alemu**

## Abstract

This study investigated students' performance and preferences on representation types as a result of multiple representations approach in calculus. The mixed method was used to carry out the pretest and posttest quasi experimental design of the non-equivalent groups. Three intact classes of size 53, 57 and 54 students from Jigjiga and Kebri-Dehar Universities in Ethiopia were the participants. The groups were taught with GeoGebra supported multiple representations approach (MRT), multiple representations approach only (MR) and conventional approach (CG). Pretest and posttest on representation type and preference were administered. The representation types were numerical, algebraic, graphical, verbal and combination. Statistically significant mean differences were obtained between the three groups on the algebraic and combination as they were determined by ANOVA ($F$ (2,161) = 6.28, $P$ = .002) and ANCOVA ($F$ (1,160), $P$ <.001), respectively, to the benefit of MRT but not on the others. The three groups demonstrated the same patterns of representation preference in the order of graphical, algebraic and verbal, and students have provided various reasons for preferring a representation type in solving calculus problem. Recommendations are forwarded that incorporating GeoGebra and further research is required to generalize to the entire population.

**Keywords:** Calculus, GeoGebra, Multiple representations, Representation preference, Representation type

## Introduction

A single representation is no longer adequate to provide a cohesive and comprehensive conceptual understanding for multifaceted features of a complex mathematical object. In other words, mathematical objects, principles, relationships, ideas, procedures, etc., can be expressed in multiple representations (MRs) including: numerical, graphical, algebraic and verbal representations (Panasuk & Beyranevand, 2011). Choosing an appropriate representation for a given task is a prerequisite for successful problem solving in mathematics (Ainsworth, 2006). This selection may entail understanding of the affordance of a particular representation in a specified context. In the principles and standards for school mathematics (National Council of Teachers of Mathematics [NCTM], 2000), there is process standard particularly referring to representations in mathematics.

These include "create and use representations to organize, record, and communicate mathematical ideas"; "select, apply, and translate among mathematical representations to solve problems"; and "use representations to model and interpret physical, social, and mathematical phenomena". Furthermore, there is a strong consensus among the mathematics education community that students can comprehend mathematical concepts using MRs (Adu-Gyamfi, Bossé, & Chandler, 2016; Adu-Gyamfi, Bossé, & Lynch-Davis, 2019; Fonger, Davis, & Rohwer, 2018; Rau, 2017). It has been ensured that students exhibit preference towards the different representations of structurally the same calculus concepts (Keller & Hirsch, 1998; Panasuk & Beyranevand, 2011; Sevimli & Delice, 2011) and students' representation variation is accounted for various conditions. Students' representation preference vary depending on the problem (Panasuk & Beyranevand, 2011), on their cognitive process (Sevimli & Delice , 2014), on the context of the problem (Keller & Hirsch, 1998), and more others. Sevimli and Delice (2014) indicate that students' preference of representation type varies according to their mathematical thinking type. To the contrary, Haciomeroglu and Chicken (2012), affirmed that cognitive abilities do not influence students' preference for visual or analytical thinking, and vice versa. As a result, the causes of variation in representation preference in learning calculus are not well spelt out. Hence, students' knowledge of and preference for MRs is prominent for deciding a strategy for problem solving. Such preference variation also accounts to varying student performance.

Student population in the Ethiopian universities is highly diversified in terms of educational, social, cultural and economic backgrounds. This can be resulted in different intellectual capacities and learning styles on the students that favor or hinder knowledge accumulation in mathematics learning. MRs in mathematics may provide the opportunity to cater wider range of students' diversity with different learning styles and modality preference. As the students get able to choose one representation in favor of the other available representations, it is highly likely that there are strong connections among representations and between representations and domain that dignify (Keller & Hirsch, 1998).

Students' representation preference is associated with various aspects of mathematics learning outcomes. It is a manifestation of the students' ability of connecting between representations and between concept and a representation (Keller & Hirsch, 1998).The advent of sophisticated computer software supports the implementation of MRs as a package augments the attention to MRs in mathematics instruction. Learning mathematics with MRs is mainly concerned with a deeper understanding (Adu-Gyamfi et al., 2016; Fonger et al., 2018; Keller & Hirsch, 1998; Rau & Matthews, 2017). The calculus curriculum is fully immersed to MRs with "The Rule of Four" to include the numerical, graphical, algebraic and verbal representations supported by technology through its reform. In the advent of computer based educational technologies, the types of representations become versatile to include simulation, animation, dynamic visualization, virtual world, augmented reality, 3D-visualization, etc.

## Students' Representation Preference in Calculus

When students have access to various representations, they have the tendency to choose one representation in favor of the other. Ainsworth (2008) indicated that students often have a preferred representation in favor of other representations for solving particular mathematical problems. Sevimli and Delice (2014) found that students' representation preference was slightly varied according to their thinking type in solving definite integral problems. The preference rate of algebraic representation decreased in the order of the thinking type: analytic, harmonic and visual, whereas the degree of preferring graphical representation was rated in the reverse order of the thinking type. Moreover, in each of the thinking type algebraic was the most preferred and numerical representation was less preferred. However, categorizing students' based on thinking type is challenging since thinking type may vary according to the context and problem type. Keller and Hirsch (1998) asserted that this preference is an essential indicator of students' connection-building between representations and between representation and domain. Durkaya et al. (2011) conducted a case study on pre-service mathematics teachers to investigate which types of representations they prefer. They used numeric and algebraic representations, but, they did not use the geometric representation. They put the justification for this situation that cognitive representation that individuals construct about a specific subject can be, even will be, different.

It is not an easy task for the students in preferring a particular representation type in favor of the other(s). Bal (2014), obtained from his study that experience and the content of the problem are the most important factors in identifying which representations to use. Being fluent in representation preference is a base for higher order thinking, and conceptual understanding and is an aspect of expertise (Fonger, 2019). To the contrary, in a small scale study, E. Haciomeroglu and G. Haciomeroglu (2020) found that students' learning preference does not affect their performance in calculus. Farihah (2018) showed that students preferred visual method and non-visual method in GeoGebra based task and paper and pen based task, respectively.

In conclusion, different students have different representation preference and their preference is influenced by different interacting factors. It is strongly believed that, when a student is able to prefer a representation among multiple representations, it is highly likely that he/she is in the position of connection building between representations as well as between representation and the concept it dignifies. These demand conducting a research on student preference of multiple representations and performance of the students as a subsequence.

**Statement of the Problem**

Calculus is one of the greatest achievements of the humans' intellectual endeavors that serve as a bridge for connecting mathematics with daily life and all other disciplines. Concepts are encoded and distributed in different representations. Even though all of the mathematical representations are meaningful for experts, some are abstract and the others are complex that demand additional cognitive efforts for novices. Despite the fact that representational fluency is a trademark for mathematical success, learners often fail to do so. In the worst cases inappropriate implementation of multiple representations can inhibit learning (Adu-Gyamfi et al., 2019; Afriyani, Sa'dijah, Subanji, & Muksar, 2018; Ainsworth, 2008; Rau, Aleven, & Rummel, 2017). Choosing an appropriate representation is an important step forward to successful problem solving (Sevimli & Delice, 2012). Gemechu, Kassa, and Atnafu (2021) found that students in Wolkite University (Ethiopia) failed to extend prior knowledge to the new concept and had difficulty to convert one representation to other in learning applied mathematics. Walelign (2014) also revealed that students at Dire-Dawa University (Ethiopia) have been joining the university with poor mathematical background.

Superfluous of MRs in mathematics classroom may not be a guarantee to confer students learning of a specific calculus concept, rather, it may lead to high cognitive demands (Ainsworth, 2006), result in representational dilemma(Rau et al., 2017) and create split-attention effect (Samsuddin & Retnawati, 2018). As a result, matching the type of representation to learning demand of students need a significantly consideration. Embedding technology into the calculus classroom may enable to use MRs as package. Hence, this study was intended to investigate the impact of multiple representation approach on first year Ethiopian university students' preference for and performance on representation type in calculus.

**Research Questions**

1. What is the impact of multiple representations approach on students' performance with regards to representation type in calculus?
2. What is the influence of multiple representations approach on students' representation preference in calculus?
3. What are the students' reasons of preferring a representation type in solving a calculus problem?

**Contribution to the Literature**

- Exploring the impact of multiple representations approach on students' representation preference and their performance with the representation types in calculus.
- Detecting students' representation preference leads to designing and incorporating multiple representations into the classroom instruction to cater the students' diverse representation preferences.
- Extending the indication of representation preference of students' ability in connecting representation with the concepts it dignifies and one representation against the other representations.

## Methods

This study aimed to investigate the impact of multiple representations instructional approach on Ethiopian university students' representation preference and performance with regard to representation type in calculus. It implemented multi-treatment pretest and post-test non-equivalent group quasi-experimental research design with mixed methods on purposefully selected groups of students that belong to two distinct universities in Ethiopia. The study compared representation preferences and performance on representation type as a result of three differentiated approaches: GeoGebra supported multiple representations approach (labeled as MRT), multiple representations approach (labeled as MR) and the conventional approach (labeled as CG) and their impact on students' performance in Calculus. Furthermore, this study investigated whether students have representation preference in favor of other representations in solving calculus problems and tried to explore associated reasons for their preferences.

### Sample and Sampling Techniques

The study was conducted on 2019/20 first year first semester students of the social science stream who got enrolled for the course mathematics for social science at Jigjiga University ( JJU) and Kebri-Dehar University (KDU). The first year social sciences students at JJU and KDU were assigned into their section (labeled as A, B, C, etc.) based on alphabetical order. Each section contains an average of 58 students (see Table 1). In the JJU and KDU, there were 23 and 12 sections of social science students, respectively. Since each of the sections were equally likely one section from JJU was assigned into the MRT (n = 53) that was taught with GeoGebra supported multiple representations approach and two sections from KDU were assigned into the MR (n = 57) that was taught with multiple representations approach and CG (n = 54) that served as a comparison group, respectively, and these were assigned randomly. The learning contents of the course that were selected for the interventions covered limit and continuity, derivatives and application of derivatives.

The MR group received multiple representations approach, with special focus on the verbal, numerical, graphical and algebraic representations. In the GeoGebra supported multiple representations treatment group, some classroom arrangement and classroom shifting was implemented during the intervention for the GeoGebra session to suitably use the computer labs for learning with the GeoGebra worksheets and for their practices with GeoGebra. The time allocation for these sessions of the groups was based on the 3 credit hours and 2 tutorial hours of the course. The students were not constrained to use the software only in the class time, but many of the students downloaded and installed in their private electronic devices. They were using it in their dormitory for practicing, experimenting and learning mathematics contents with their own peace. GeoGebra was implemented in two approaches.

On one hand, there were a wide ranging collection of calculus related dynamic and interactive worksheets available online in a downloadable form that were presented and demonstrated in the classroom and provided for the students to practice in the computer lab. On the other hand, the course instructor was preparing such dynamic and interactive contents on the fly during the lesson depending on the students' reaction. The latter approach was helpful to create a flexible and reflective teaching style. In the MRT group, by means of GeoGebra, the teaching and learning of calculus was shifted to be more active and dynamic where students explored calculus concepts with linked multi-representations, which is often difficult using chalk and board. The CG was taught based on the conventional approach, which was more dominantly algebraic representation. The intervention lasted for about six weeks.

Table 1. Participants Background Information

| | | Gender | | | |
|---|---|---|---|---|---|
| | | Female | | Male | |
| Group | N | f | % | f | % |
| MRT | 53 | 16 | 30.2 | 37 | 69.8 |
| MR | 57 | 21 | 36.8 | 36 | 63.2 |
| CG | 54 | 18 | 33.3 | 36 | 66.7 |

**Data Collection Instruments and Procedures**

The data collection instruments of this study included open ended tests, Preference Questionnaire, representation preference reasoning problems and classroom observation. Since tasks that involve mathematical problems based on MRs can lend themselves to rubric assessment and to other assessment types suitable for open-ended activities, the students' score on the representation type was quantified using rubric assessment technique. The test items were prepared based on different constructs that had their own rubric for assessing and scoring results.

The scores were categorized into representation type for the purpose of this study. The pretest was compiled from the contents of pre-calculus (i.e., functions) and the posttest was compiled from the contents covered in the intervention (i.e., limits, continuity, derivatives and application of derivatives). These tests were collected and adapted from different calculus textbooks and other relevant sources from the literature (*e.g.*, Keller & Hirsch, 1998).

To determine students' representation preference, representation preference inventory (RPI) was administered. For RPI, five optional representations for a given particular mathematical problem were provided as response choices (verbal, numerical, graphical, algebraic, and combination). In the RPI, students were asked to write the reason of their choice for a specific type(s) of representation(s) from the given options. The RPI was intended to determine the modes or types of representation(s) that students choose in communicating, justifying, reasoning and explaining their mathematical problem solving paths. The students were free to choose more than one types of representations and they were asked to provide a reason for the representation(s) type(s) of their choice(s). The students' reason for choosing the particular representation type was coded and categorized.

To understand the students' cognitive processes during interaction with the representations, the pretest and posttest were prepared based on the constructs under representation translation, representation interpretation and representation implementation. For the purpose of this study, the students' score on each item was categorized based on representation type and the three groups were compared on these scores. The classroom discourses were captured through observation checklists during a lesson.

The information obtained from the classroom observation was used to triangulate the results that were obtained from analysis of the data collected by the different data collection tools. Each group was observed three times during the lesson delivery. An attempt was made to elucidate how the MRs enhanced classroom interaction and disciplinary discourse among the students.

**Reliability and Validity of Instruments**

With the intention of obtaining reliable and valid information from the data collection instruments, several efforts have been made. The mechanisms to ensure validity included face validity, content validity, construct validity and criterion validity. In order to establish validity of the data collection instruments for the different constructs of the representation type(s) comments from colleagues were used and the items were modified accordingly. In addition to the colleagues' comments, the opinions of mathematics experts, who are member of the academic staff in mathematics department at JJU, were consulted to check the validity of a concept and appearance from the aspects that it aimed to measure. The assessors also evaluated the appearance of the items in each of the constructs in terms of feasibility, readability, consistency of style and formatting, and the clarity of the language used to the level of the participants experience. Panel of experts were also involved to evaluate content validity of the constructs to ensure whether each of the constructs incorporates all the items that were essential and they eliminated irrelevant items in any of the constructs. In addition to the panel of experts, the literature review was used to establish the validity of the constructs.

To establish the reliability of the instruments of each construct, a pilot test was conducted on second year mathematics department students at JJU. Two iterators were involved in assessing the students' work using the predetermined rubrics for scoring students' solution of the items in each of the constructs. As a result of this, students' solution were analyzed separately by two mathematics department academic staff members at JJU and the calculation of the reliability were computed manually using the formula "Consensus/ (Consensus + Dissensus) X 100" ( Miles & Huberman , 1994, as cited in Bal, 2014, P.2354). Cronbach's alpha was used to assess the internal consistency reliability of the RPI. Based on the reliability values displayed in Table 2, all of the constructs were found to be in the acceptable range of reliability (Berry & Mielke, 1988).

Table 2. Reliability Values of the Instruments

| Instrument | Component | Reliability | |
|---|---|---|---|
| | | Pretest | Post-test |
| Representation Type Tests | Numerical Representation | .81 | .75 |
| | Graphical Representation | .81 | .68 |
| | Algebraic Representation | .78 | .77 |
| | Verbal Representation | .73 | .76 |
| | Combining Representation | .74 | .78 |
| | Representation Preference | .69 | .67 |

**Data Analysis**

To compare the three groups with regards to their performance on the representation type appropriate inferential statistics were used depending on the natures and properties of the collected data. Parametric and Non-parametric statistical tools that were appropriate for comparing the three groups on the representation type such as one-way ANOVA and ANCOVA or Kruskal –Wallis were used depending on the normality of the data. The three groups were also compared on the representation preference using percentage and frequency. Similarly, the qualitative data were analyzed using the category of codes and theme techniques. Preliminary analyses were done on the data sets to inspect the nature of the collected data using descriptive statistics and graphical presentations. This activity paved the way for choosing the correct and appropriate inferential statistical techniques to address the research questions.

# Results

## Pre Intervention Results

To determine the base line difference of the three groups on the focused learning outcomes related to representation preference and performance on the representation types in learning calculus concepts, a pretest was administrated before the intervention had been begun. The outcomes of one way ANOVA reported in Table 3 show that there were no significant difference among the three groups on the Algebraic ($F$ (2,161) = 1.50, $P$ = .227) and the Verbal ($F$ (2,161) = .54, P = .585).

However, significant differences were obtained among the three groups on the Numerical ($F$ (2,161) = 10.06, P < .001), on the Graphical ($F$ (2,161) = 5.29, $P$ = .006) and the Combination ($F$ (2, 161) = 13.21, $P$ < .001). These results manifested that students in the three groups had demonstrated variations in their performance in the indicated representation types. Hence, the Numerical, Graphical and Combination were considered a covariate for the corresponding posttest.

Table 3. One-way ANOVA Results on the Representation Types Pretest

| Representation type | Sources | *SS* | *Df* | *MS* | *F* | *P* |
|---|---|---|---|---|---|---|
| Numerical | Between Groups | 666.84 | 2 | 333.42 | 10.06 | .000 |
| | Within Groups | 5336.92 | 161 | 33.15 | | |
| | Total | 6003.76 | 163 | | | |
| Algebraic | Between Groups | 164.69 | 2 | 82.35 | 1.50 | .227 |
| | Within Groups | 8858.06 | 161 | 55.02 | | |
| | Total | 9022.75 | 163 | | | |
| Graphical | Between Groups | 473.84 | 2 | 236.92 | 5.29 | .006 |
| | Within Groups | 7226.60 | 161 | 44.89 | | |
| | Total | 7700.44 | 163 | | | |
| Verbal | Between Groups | 6.80 | 2 | 3.40 | .54 | .585 |
| | Within Groups | 1017.90 | 161 | 6.32 | | |
| | Total | 1024.70 | 163 | | | |
| Combination | Between Groups | 386.31 | 2 | 193.16 | 13.21 | .000 |
| | Within Groups | 2354.20 | 161 | 14.62 | | |
| | Total | 2740.51 | 163 | | | |

Based on the information provided in Table 4, the most preferred representation type of the MRT was Algebraic (26 (49.1%)), followed by Graphical (20(37.7%)) and Verbal (7 13.2%). More than half of the students in the MR preferred Algebraic (41(71.9%) with little interest in the Graphical (9(15.8%)) and the Verbal (7(12.3%). Likewise, the most preferable representation type of the CG was the Algebraic (31 (57.4%)) followed by the Graphical (16(29.6%) and the Verbal (7(13.0%). From these, it can be conclude that students who belong to the three groups had similar pattern of representation preferences before the intervention in which algebraic representation was the most popular mode of representation.

Table 4. Representation Preferences pretest Frequency and Percentage

| Group | N | Numerical | | Graphical | | Algebraic | | Verbal | | Combination | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | f | % | f | % | f | % | f | % | f | % |
| MRT | 53 | - | - | 20 | 37.7 | 26 | 49.1 | 7 | 13.2 | - | - |
| MR | 57 | - | - | 9 | 15.8 | 41 | 71.9 | 7 | 12.3 | - | - |
| CG | 54 | - | - | 16 | 29.6 | 31 | 57.4 | 7 | 13.0 | - | - |

Note: the column groups Numerical through Combination are printed under the spanning heading "Representation Preference".

**Post Intervention Results**

The contents of the post tests were compiled from the contents that were covered during the intervention (limits, continuity, derivatives and applications of derivatives) that had similar constructs and conceptual structures with the pretest instruments. A preliminary data analysis was done on the representation types to check for any violation of the assumptions of the underlying statistical techniques that were used to address the research questions and some related issues raised from the data set.

*Results of Representation Type Posttest*

The representation types that were considered in this study included: numerical, algebraic, graphical, verbal and combination extracted from problems of different constructs posed in these representation types.

> **H $_{[O1]}$:** There are no statistically significant mean differences among the three groups on the representation types posttest.

The means and standard deviations of the representation types posttests were more or less similar across each of the groups, except on the combination at which the mean score of the MRT (*M* = 18.77, *SD* = 3.27) was slightly larger than the MR (*M* = 16.35, *SD* = 3.19) and the CG (*M* = 17.15, *SD* = 3.11). But, from this descriptive data it is too early to infer on relative positions of the three groups with respect to representation types until running the appropriate inferential statistics (see Table 5). All, but the graphical representation type, had skewness in the range of -1 and +1 and the skewness Z-values were not beyond +/- 1.96. These results revealed that the numerical, algebraic, verbal and combination representation type were approximately normally distributed. However, the skewness Z- value of the graphical representation for the CG was beyond the range of +/- 1.96 and the graphical representation in the CG was not normally distributed.

Table 5. Descriptive Statistics of Representation Types Posttest

| Group | N | Numerical | | Algebraic | | Graphical | | Verbal | | Combination | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *M* | *SD* | *M* | *SD* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| MRT | 53 | 19.43 | 4.60 | 20.98 | 3.20 | 24.83 | 6.42 | 11.72 | 2.14 | 18.77 | 3.27 |
| MR | 57 | 19.30 | 3.08 | 19.04 | 2.90 | 24.93 | 4.81 | 11.05 | 2.51 | 16.35 | 3.19 |
| CG | 54 | 19.20 | 3.48 | 19.44 | 2.14 | 25.41 | 5.20 | 11.74 | 2.29 | 17.15 | 3.11 |

Since Algebra and combination were not considered to be covariates, one way ANOVA was conducted (see Table 6). This study was intended to infer students' performance on the representation types to accomplish a mathematical task and to identify the affordances and obstacles associated with particular kinds of representation as a result of multiple representations approach. A one way between groups ANOVA was conducted to compare the three groups on the Algebraic and Verbal representation type posttests. A statistically significant mean difference were obtained on the Algebraic ($F$ (2, 161) = 6.28, $P$ = .002, $\eta^2$ = .07) between the three groups. The result of $\eta^2$ indicated that 7% of the variances of the three groups on the Algebraic was explained by the treatment type. This effect size was in the range of medium to large (Berry & Mielke, 1988). Based on the output in Table 6, no statistically significant mean differences were obtained on the Verbal ($F$ (2,161) = 1.58, $P$ = .210, $\eta^2$ = .02). A post-hoc comparison using Tukey HSD test indicated that the mean score of the MRT was significantly greater than the MR ($P$ = .003) and the CG ($P$ = *.025*) on the Algebraic, but the output in Table 7 revealed that the MR did not significantly differ from CG on the Algebraic ($P$= .755).

Table 6. One way ANOVA Results of Algebraic and Combination Posttests

| Variable | Source | *SS* | *df* | *MS* | *F* | *P* | $\eta^2$ |
|---|---|---|---|---|---|---|---|
| Algebraic | Between Groups | 114.12 | 2 | 57.06 | 6.28 | .002 | .07 |
| | Within Groups | 1462.24 | 161 | 9.08 | | | |
| | Total | 1576.36 | 163 | | | | |
| Verbal | Between Groups | 17.03 | 2 | 8.51 | 1.58 | .210 | .02 |
| | Within Groups | 869.97 | 161 | 5.40 | | | |
| | Total | 886.99 | 163 | | | | |

Table 7. Multiple Comparisons on the Algebraic Posttest

| Variable | (I) Group | (J) Group | *MD* (I-J) | *SE* | *P* |
|---|---|---|---|---|---|
| Algebraic | MRT | MR | 1.95* | .58 | .003 |
| | | CG | 1.54* | .58 | .025 |
| | MR | CG | -.41 | .57 | .755 |

*: The mean difference is significant at the 0.05 level.

For the other three representations, one way ANCOVA was performed for numerical and combination that fulfilled normality and Kruskal-Wallis for graphical the results of which are presented in Table 8 and Table 9.

Table 8. ANCOVA Results of Numerical and Combination Posttests

| Source | *SS* | *df* | *MS* | *F* | *P* | $\eta^2$ |
|---|---|---|---|---|---|---|
| Numerical Pretest | .749 | 1 | .749 | .073 | .787 | .000 |
| Group | 1.389 | 2 | .695 | .068 | .934 | .001 |
| Error | 1632.959 | 160 | 10.206 | | | |
| Combination Pretest | 10.217 | 1 | 10.217 | 1.004 | .318 | .006 |
| Group | 175.207 | 2 | 87.603 | 8.605 | .000 | .097 |
| Error | 1628.863 | 160 | 10.180 | | | |

A one way ANCOVA was used to assess whether the three groups had variation on the Numerical and Combination post-tests after the Numerical and Combination pre-tests had been controlled. After adjusting for pre-intervention scores, there was no significant difference between the three groups on post-intervention scores on the Numerical ($F$ (1, 160) = .068, $P$ = .934, $\eta^2$ = .001). As it is evident from this result, the effect of treatment type on the Combination posttest remains significant ($F$ (1, 160) = 8.605, $P$ <.001, $\eta^2$ = .097) after the variation on the Combination pretest had been controlled.

The data obtained from the graphical posttest did not meet the stringent assumptions of the parametric techniques. The Kruskal-Wallis test was used since the assumption of normality was violated. The result in Table 9 evidenced that there was no significant mean rank difference between the three groups on the graphical representation posttest ($\chi^2$ (2) = .542; $P$ = .762). This result revealed that students in the three groups did not vary according to the treatment type they received regarding the graphical representation type.

Table 9. Kruskal-Wallis Test Result on the Graphical Posttest

| Variable | Group | N | Mean Rank | $\chi^2$ | *df* | *P* |
|---|---|---|---|---|---|---|
| | MRT | 53 | 83.67 | .542 | 2 | .762 |
| Graphical | MR | 57 | 78.89 | | | |
| | CG | 54 | 85.17 | | | |

From the results provided in the previous successive tables, no evidence was gained to accept the null hypotheses regarding the algebraic and combination, but regarding the Numerical, Graphical and Verbal Representation Types the null hypothesis is accepted.

*Results of Representation Preference*

With the advancements of computer technologies, students' access to the package of MRs of mathematical concepts has increased. Whenever students have cognitive preference for a particular representation type in favor of the other, they may have developed strong connections with representations and concepts. Hence representation preference is key factor for determining prominent learning outcomes.

**$H_{[O2]}$: T**here is no difference among the three groups on representation preference after the intervention.

Table 10. Frequency and Percentage of representation preference

| | | Representation Preference | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Numerical | | Graphical | | Algebraic | | Verbal | | Combination | |
| Group | N | *F* | % | *f* | % | *f* | % | *f* | % | *f* | % |
| MRT | 53 | - | - | 37 | 69.8 | 16 | 30.2 | - | - | - | - |
| MR | 57 | - | - | 28 | 49.1 | 23 | 40.4 | 4 | 7 | 2 | 3.5 |
| CG | 54 | 2 | 3.7 | 29 | 53.70 | 20 | 37.0 | 3 | 5.6 | - | - |

$\chi^2(8) = 14.357$, N = 164, $P$ = .073

The representational preference was a multiple choice questionnaire type provided with the problem context and the students were required to choose one or more representation type(s) to solve the given problem. When the students choose more than one, it was labeled as combination. The multiple-choice questionnaire were presented with tasks represented in numerical, graphical, algebraic and verbal modes and the students were requested to indicate the solution method they would prefer to accomplish the given task: ‘use the table’ , ‘use the graph’ , ‘use the algebraic’, ‘use the verbal’ and ‘use the combination’(Keller & Hirsch, 1998). The students did not do any calculation. They were also requested to write their reason of choosing a particular representation type. An effort was made to make each representation as visual as possible and to ensure that each representation could be used to answer each question with equal ease. Frequency and percentage were used to identify students who preferred a particular representation type. Based on the outcomes displayed in Table 10, regarding representation preference , the most preferable type of representation for the MRT was graphical (37 (69.8%)) followed by algebraic (16 (30.2%)). The type of representation that was most preferable for the students in the MR was graphical (28 (49.1%)) followed by algebraic (23 (40.04%)) and Verbal was the least preferred (4 (7%). Similarly, for the CG, the most preferred representation was graphical (29 (53.70%)) followed by algebraic (20 (37.0%). Even though, the percentages of students in each group with respect to the type of representation preference vary, the prevalence of the representation type has the same degree and pattern in each group. Graphical representation was the most popular type of representation with most favorable for the MRT group. Based on this information, it can be concluded that students who belong to the three groups demonstrated slight variations on their preference of representation type. After the intervention, the students in the MRT group sharply shifted their preference from algebraic (49.1%) representation towards graphical representation (69.8%) (see Table 4 & Table 10).

The qualitative component of the information provided another layer of evidence for the student’s reason of preferring a particular representation type. Students’ justification for their reason of preferring a particular representation type in favor of other representations had the power of informing why they preferred a particular type of representation. Students, who came from the MRT, were trying to use graphical representation to solve the problems. It was also witnessed in the classroom observation that students were tried sketch graphs to demonstrate the calculus contents using GeoGebra.

**Students’ Reasons for Representation Preference**

Students had various justifications for their preference of a representation type. The students’ reason for representation preference was analyzed based on each representation type because there is a possibility for them to prefer different representations in different contexts for various reasons.

*Numerical*

The theme that emerged for the students’ reason for numerical representation preference was the phrase “*seeking exact value*”. In the RPI posttest, item Q3 was concerned with the phenomenon that the height of a plant (in meters) after *t* years since the time at which it was first planted is given in the table, graph, equation and verbal explanation. To find the instantaneous rate of growth of the height of the plant at t = 2, the students were asked to tell whether they prefer to use: (A) the graph (B) the table of values (C) the equation (D) the verbal explanation. The students' preference of the representation type to solve this problem had great variation and the justification of preferring a representation type for solving this particular problem was immense. The justification of a student who preferred the numerical representation type was seeking exact value from the table value. Students' sought *exact value* using numerical representation, as the verbatim from a student of the MRT below dignifies:

> *Using the table is better than others because from the table we get the exact height of the plant at t = 2. When we see the graph it shows the rate of growth well but to find the height of the plant at specific point is difficult. When we take the equation we can get the exact value but it will take some process and we will make a mistake when we calculate and get wrong value. When we take verbal expression it shows the increment and decrement of the tree only but does not show at which year the tree grows faster or slowly* (S1MRT: February 19, 2020).

A justification wrote up from the MR group stated as:

> *The table value is preferable to find the height at specified value of time, we get the height at t = 20sec is 35cm but other expressions are difficult to analyze or get the exact value at distinct point* (S1MR: February 19, 2020).

Another item seeking the behavior of the function $f(x) = sin\frac{1}{x}$ was described by the table of values, graph, algebraically and verbal explanations. The students were asked which to prefer to estimate: $\lim_{x\to 0} sin\frac{1}{x}$ . In response to this a student of the CG mentioned that:

> *The table is preferable to estimate the limit of f(x). The table shows the limits of f(x) that oscillate between -1 and 1 and it shows the limit at x = 0. The value of f(x) does not exist directly and the graph shows better for the oscillation but it is difficult to analyze the limit at x = 0 and also other expression shows about the oscillation of the value of f(x) from the left and right at x = 0 but they did not show the limit at x = 0* (S1CG: February 19, 2020).

In line with this result, Keller and Hirsch (1998), found that all students in an interview shared a comments that a table of values give exact answer but may skip the value of interest, graphs often do not provide a precise value; and working with equations has the most potential for errors.

*Graphical*

One of the themes that emerged for the justification of graphical representation preference was that it "*provides sufficient information".*

> *"I prefer the graphical representation because graph gives sufficient information"* (S2MRT: February 19, 2020).
>
> *The path of the point continuously fluctuated because of the oscillation to increase or decrease (*S3MRT: February 19, 2020).
>
> *The graph illustrates that the rate of growth of the tree was very slow around the beginning, however, as time elapsed, the tree grows very rapidly. In the graph, it is observed that the tree growth by the rate of 20 for every unit of time t* (S2MR: February 19, 2020).

> *By using the graph we can identify the change of height, the graph shows the change of height at which interval of time it grows faster and at which interval of time the height grows quickly* (S2CG : February 19, 2020).

Based on these sample students' justifications for their reason of choosing the graphical representation in solving calculus problems, it can be confirmed that students prefer graphical representation due to its sufficient information density.

*Algebraic*

Some students preferred equation because of "*ease of substituting*". This indicates that the students preferred algebraic representation for procedural purpose.

> *I would prefer using the formula because it guarantees me to do so every equation (*S4MRT: February 19, 2020*).*
>
> *I prefer to use the formula because we can find this area of a circle from the formula simply by substitution (*S3MR: February 19, 2020*).*
>
> *The equation is preferable than other expressions to find the instantaneous velocity. To find the instantaneous velocity we can substitute t = 3 directly into the equation and get the required value easily. The graph is difficult to analyze the exact value. The table gives the exact but we can't know how we get this value, when we see the verbal it is not insightful to find value. It tells the form of the function but does not give the form of the function and the exact value (*S3CG: February 19, 2020*)*

These results justified that students' preference of algebraic representation for solving calculus problem was for the procedural purpose. They were confortable to solve the problem by substituting to the explicitly formulated algebraic formula using the algebraic rules and procedures. This result contradicts with finding of Neria and Amit (2004) that reports higher achievers choose to communicate mathematical solutions via algebraic representation.

*Verbal*

The theme that emerged for analyzing the students' justification of their reason for selecting verbal representation to solve calculus problems was verbal due to its "*Expressive*" power. Some of the notes indicated by the respondents were:

> *I would prefer the verbal explanation because it makes me aware when the function gets close to zero, since it is given. It is unnecessary to do more and take time (*S5MRT: February 19, 2020*).*
>
> *Verbal explanation is preferable than others because it shows the behavior of the function better than others. The limit of f(x) when $x \to 0$ is 2 but at x = 0 the function undefined. When we see the graph at x = 0, it must be a hole. The equation did not show the limit value and the table value did not show the value of the function at x = 0* (S5MR: February 19, 2020).

Words are ubiquitous and are used to represent ideas and relations inside and outside the domains of math and science. Words are also very expressive, and facility with words helps people to communicate their mathematical ideas and understand the ideas of others (Nathan & Kim, 2007). Verbal explanation has the potential to contextualize the complex problem in which the others can reduce and simplify to represent.

*Combination*

One of the themes that emerged for combination and the reason for preferring the combination to various representations is the "*Complementary*" nature of two or more representations. The excerpt below indicates this.

> *Since the graph is sketched according to the value of the table both of them give clues to predict the time at which the ball reaches to the ground* (S6MRT: February 19, 2020).

From these justifications for the representation preference, it can be observed that students preferred various representations for various reasons. Students could more clearly elaborate on their justification for using certain representations. Choosing an appropriate representation is an important step to successfully solve a problem. It includes decisions about what is taught and how it is taught including, what representations are (intentionally or unintentionally) preferred and ignored. Privileging of one representation reflects the students' linking of the concept with a particular representation type.

The students, who came from the MRT groups, used GeoGebra installed in their smart phones to solve a problem using graphical representation. Despite the fact that students preferred graphical representations to perform a mathematical task, they used algebraic and verbal representations in practice. Students were struggling to sketch graph of a function since they failed to understanding the basic information required to sketch a graph. The students' justification for their reason of preferring a particular type of representation is not consistent across each representation and group. This may lead to the conclusion that students' representation preference depends on the nature of the problem and the representation type.

## Discussion

The students' results obtained from the representation type posttest data were extracted from the items under different constructs posed in different representation types. Students are more successful in problem solving using a specified representation type and their success was strongly influenced by the representation format. The representation types that were considered in this study included: numerical, algebraic, graphical, verbal and combination. The items were extracted from the constructs of representation interpretation problem, representation implementation problem and representation translation problem posed in any of the representation type.

A one way between groups ANOVA was conducted to determine the impact of multiple representations approach on students' performance on the representation types of calculus concepts. A statistically significant mean difference were obtained on the Algebraic ($F$ (2, 161) = 6.28, $P$ = .002). No statistically significant mean differences were obtained between the groups on the Verbal ($F$(2,161) = 1.58 , $P$ = .210 ) and the Graphical

($F$(2,161) = .17 , $P$ = .844). A post-hoc comparison using Tukey HSD test indicated that the mean score of the MRT ($M$ = 20.98, $SD$ = 3.20) was significantly higher from the MR (M = 19.04, SD = 2.90) and the CG ($M$ = 19.44, $SD$ = 2.94) on the Algebraic. The MR did not significantly differ from CG on the Algebraic. This result coincides with Dreher and Kuntze (2015).

ANCOVA was used to assess whether the three groups had variation on the Numerical and Combination post-tests after the Numerical and Combination pre-tests had been controlled. After adjusting for pre-intervention scores, there was no significant difference between the three groups on post-intervention scores on the Numerical ($F$ (1, 160) = .068, $P$ = .934). The effect of treatment type on the Combination posttest remains significant ($F$ (1, 160) = 8.605, $P$ <.001). The MRT was more benefited than the MR; even the MR was less advantageous than the CG on the combination of representations.

The data obtained from the graphical post-test did not meet the demanding assumptions of the parametric techniques. The Kruskal-Wallis test was used since the assumption of normality of distribution was violated. There was no significant mean rank difference among the three groups on the graphical representation post-test ( $X^2$ (2 ) = .542, $P$ = .762). This result revealed that students in the three groups did not vary according the treatment type they received regarding the graphical representation. Graphs take on particular importance because they appear in a variety of fields outside mathematics, particularly in the physical and social sciences, where they are used to represent data and express theoretical relationships. This result may coincide with the discovery of E. Haciomeroglu and G. Haciomeroglu (2020) that indicates the mismatch between instructional mode and learning preference did not affect the students' performance in calculus.

The availability of multiple representations within the mathematics classroom can change students' perception regarding the acceptable form of solution to a problem. Furthermore, Mielicki and Wiley (2016) affirmed that both problem type and representation format affected problem solving performance. Problem type and representation type interact to affect students' success in problem solving. The students' justification for their reason of representation preference showed that they preferred algebraic representation and graphical representation for computation problem and for interpretation problem, respectively.

The Representational Preference Inventory (RPI) posttest was a multiple choice questionnaire type provided with the problem context and the students were required to choose one or more than one representation type to solve the given problem. When the students chose more than one, it was labeled as combination. The multiple-choice questionnaire were presented with tasks represented in numerical, graphical, algebraic and verbal modes and the students were requested to indicate the solution method they would prefer to accomplish the given task: 'use the table' , 'use the graph' , 'use the algebraic', 'use the verbal' and 'use the combination'(Keller & Hirsch, 1998). In this way, the participants were expected to identify the representation type which they believed would facilitate the process of solving a given problem (Sevimli & Delice, 2011). The students did not do any calculation. They were also requested to write their reason of choosing a particular representation type. An effort was made to make each representation appear equally likely and to ensure that each representation could be used to answer each question with equal ease. The results obtained from frequency and percentage showed that the most preferable type of representation for the MRT was graphical representation (37 (69.8%)) followed by algebraic representation ( 16 (30.2%)). This result is in line with result of Keller and Hirsch (1998) that indicates students who used graphic calculators as tools were more likely to have a graphical representation preference. Farihah (2018) stated that students who have different learning styles and were taught with the support of GeoGebra prefer solving calculus problems using graphical representations. Likewise,Sevimli and Delice (2014) indicate that technology influence students' preference of representation type in calculus. The type of representation that was most preferred for the students in the MR was graphical (28(49.10%)) followed by algebraic (23 (40.40%)) and Verbal was the least preferred (4 (7%). Similarly, for the CG, the most preferred representation was graphical (29 (53.7%)) followed by algebraic (20 (37.0%) and numerical (2(3.7%). Mielicki and Wiley (2016) indicated that that graphical representation facilitates problem solving among college students. Even though, the percentage of students in each group with respect to the type of representation preference had variations, the prevalence of the representation type has the same degree in each group. Graphical representation was the most popular type of representation with most favorable for the MRT group. Based on this information, it can be conclude that students who belong to the three groups demonstrated slight variations on their preference of representation type. The percentage of students who preferred graphical representation was increased by 32.1% for the MRT, 33.3% for the MR and 24.1% for the CG. These results revealed that the treatment type equally likely influenced for both the MRT and the MR, but it was less for the CG regarding their preference towards graphical representation.

After the intervention, the students in the MRT and the MR groups sharply shifted their preference from algebraic representation towards graphical representation, and their proportion dropped by 18.9% and 31.5%, respectively. It is clear that the students in the MRT group made a tremendous shift from algebraic preference to graphical representation after the intervention. Hamid, Idris, and Tapsir (2020) showed that students were very encouraged to use graphs in the teaching and learning of mathematics but the opposite was evident in their worked solutions. Selecting an appropriate representation for a given task is a critical part of successful problem solving, and this selection entails understanding the affordances of different representations in different contexts (Ainsworth, 2006).

The qualitative component of the information provided another layer of evidence for the student's reason of preferring a particular representation type. Students' justifications of reasons for their representation preference were fragmented and categorizing these into all-encompassing themes that can encapsulate all the reasons was a little bite cumbersome. Students had various justifications for their preference of representation type. To overcome this change, the reasons were categorized based on the representation type. As a result, different themes emerged for the reasons of the students preferring one representation(s) in favor of the other representations in solving a particular calculus problem.

The theme that emerged for the students' reasons of numerical representation preference might be phrased as "*seeking exact value"*. The justification of students who preferred the numerical representation type was seeking exact value from the table value. In line with this result, Keller and Hirsch (1998), found that all students in an interview shared comments that a table of values give exact answer but may skip the value of interest, graphs often do not provide a precise value; and working with equations has the most potential for errors. Students' justification for preferring graphical representation phrased as "*provides sufficient information"* was confirmed by the reasons that graph gives sufficient information. Some students preferred algebraic representation (or equation) because of "*ease of substituting"*. This indicates that the students preferred algebraic representation for procedural and computational purposes. Students used algebraic representation to solve problems by "plug in- plug out" method. The students put in the sample problem as "Use the formula because we can find area of a circle from the formula simply by substitution". Students preferred verbal representation because of its *expressive* power. Words are ubiquitous and are used to represent ideas and relations inside and outside the domains of mathematics and science. However, words have always been important to mathematics and mathematics education, in part, because verbal representations can carry ideas across disciplines. Words are also very *expressive*, and facility with words helps people to communicate their mathematical ideas and understand the ideas of others (Nathan & Kim, 2007). Verbal explanation has the power of contextualizing a mathematical concept without reducing and simplifying its natural setting. Some students preferred the combination of various representations for *Complementary* function of the component representations. The students explained that since the graph is sketched according to the value of the table, both of them give clue to comprehend the concept.

From these justifications for the representation preference, it can be observed that students preferred various representations for various reasons. Students could more clearly elaborate on their justification for using certain representations. Privileging of one representation reflects the students' linking of the concept with a particular representation type and their ability of linking the different representations.

The classroom observation paved the way to capture the live experience of the classroom atmosphere for all tacit and explicit behavior of the classroom and students' interaction. The purpose of the classroom observation in this study was to achieve a holistic understanding of the student interaction with the multiple representations in classroom for triangulating the results. The main focus of the classroom observation was to get an insight to the emerging dynamics that were evoked by the use of multiple representations including capturing nuances of students' thinking processes while engaged in problem solving using these representations. The results obtained from the classroom observation checklist revealed that students in the GeoGebra supported multiple representations approach frequently exited students in drawing a graph of a function using GeoGebra in collaboration with their peers during the computer lab session. Students used verbal explanation to explain mathematical ideas in the classroom discourse and they did not support the graphical representation with written text. However, the multiple representations method of instruction classroom, the students failed the basic information of a function (intercepts, domain and range, symmetry, interval of increasing and decreasing, and so on) to illustrate the calculus of a function using graphical representations. The MRT often focused on sketching graphs using GeoGebra to help them to make conjectures, analyze or justify their solutions.

## Conclusion

The GeoGebra supported multiple representations approach was more supportive for analytically (algebraically) solving calculus problems than the multiple representations method of instruction and the conventional approach. Moreover, the GeoGebra supported group benefited more from the combined representations in performing calculus tasks. Graphical representation was the most popular type of representation that was most favorable for the MRT group. It can be concluded that GeoGebra and similar utilities are helpful for generating extensive representation preferences since they support to implement the full "package" of multiple representations.

**Recommendations**

Hence, the following bits of recommendation were forwarded.

- Mathematics course instructors must differentiate their students to cater their diversified representation preferences.
- Academic officials, course instructors and students have to integrate GeoGebra into their calculus classroom instruction to implement the full "package" of multiple representations.
- A further research need to be conducted to identify the association between the students' preference of representation type and the tendency of implementing the representation of their preference in solving a problem as well as their effectiveness on the preferred representation type.
- A further research is required to determine the interacting factors that can influence students' representation preference in solving a calculus problem.

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