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Probabilistic Thinking for Life: The Decision-Making Ability of Professionals in Uncertain Situations

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Abstract

Although most experts in the field believe that the study of probability would equip people in making good decisions, they have overlooked the need to examine professionals' decision-making ability in their daily activities after many years of probability studies. This study explored four non-mathematics teachers who studied probability at High and Primary schools. It appears no one has written about this topic. Despite this scholarly neglect, this study explains the need for research in the area and offers an initial interpretation of this new area. The study contends that understanding people who studied probability at least 13-years ago could provide us with insight on how people use probability in making decisions on uncertain situations and to inform the teaching of probability. Content analysis was done on the recorded data. The participants' response to tasks and questions revealed their inability to apply probabilistic skills in uncertain situations. The participants appear to be either overconfident or making choices out of habit, without a critical assessment of the various possibilities and their consequences. This study calls for a 'cross-examination approach' in the development of probabilistic thinking skills among students.

Introduction

Probabilistic situations are experienced daily and in professional life. Daily and professional life aspects are an important reference point for instruction (Ozmen & Baki, 2021). Statistical and probabilistic literacy is a necessary skill of expected outcome of schooling (Gal, 2002). People with statistical and probabilistic literacy interpret, critically evaluate, assess, analyze, judge, decide and communicate about uncertain situations. Day in day out, we encounter uncertain situations in which we must make a choice. People make a judgment "if they can trust someone, if we should do something (or not), which route to take, how to respond to someone's question" (Dale, 2015, p.1). Even animals cannot escape probabilistic situations. They make choices on how to live to be safe from other dangerous animals when they have not yet seen those dangerous animals. So animals act about future possibilities which they are uncertain about. If there was a school in the animal world, one mathematical concept which would be given the needed attention in their curriculum would be probability. Every country is concerned with promoting statistical and probabilistic literacy skills among its citizens. So, most countries include probability in their mathematics curriculum (Hokor & Sedofia, 2021). This is because probability empowers people to make good choices. Whether we choose or not there is a consequence for our decision. This consequence may be good

or bad depending on the choices we make. Our knowledge of statistical and probabilistic reasoning provides us with the necessary skills in making decisions by understanding the uncertain nature of life (Batanero et al, 2016). Statistical and probabilistic reasoning is life, for life without statistical and probabilistic reasoning skills is like a dry leaf that can easily be blown away by the wind (Amir & Williams, 1999). In this century and the century to come, the importance of statistical and probabilistic reasoning remains incontestable (Gal, 2002). This is because we continue to meet situations in which probability and statistics play important roles.

Probability is the study of randomness and uncertainty. It is the numerical estimation of a possibility that something may or may not happen under uncertainty. Probability can also be seen as numerical quantification of the occurrence or non-occurrence of an event under uncertainty. Randomness is the lack of pattern or predictability in events or when a sequence of events has no order and does not follow a reasonable pattern. Random events cannot be predicted with certainty. For example, the movement of subatomic particles is random and needs to be analyzed in terms of probability. Uncertainty is a situation that involves unknown facts. It is concerned with predictions of future events which has elements of doubt. An event is an outcome or set of outcomes of an experiment. Also, uncertainty is where some possible outcomes have an undesired effect or significant loss. The reasoning about uncertainties is termed probabilistic reasoning. Hokor (2020) refers to probabilistic reasoning as the "way people reason about uncertain situations and make decisions based on the likely outcomes" (p. 2). Bryant and Nunes (2012) asserted that "this reasoning allows us to work out the probability of particular outcomes, and thus to understand the risks and possible benefits of acting in one way rather than another" (p.3). The ability to identify the most likely outcomes on uncertainty constitutes probabilistic thinking. Figure 1 presents a model for probabilistic thinking.

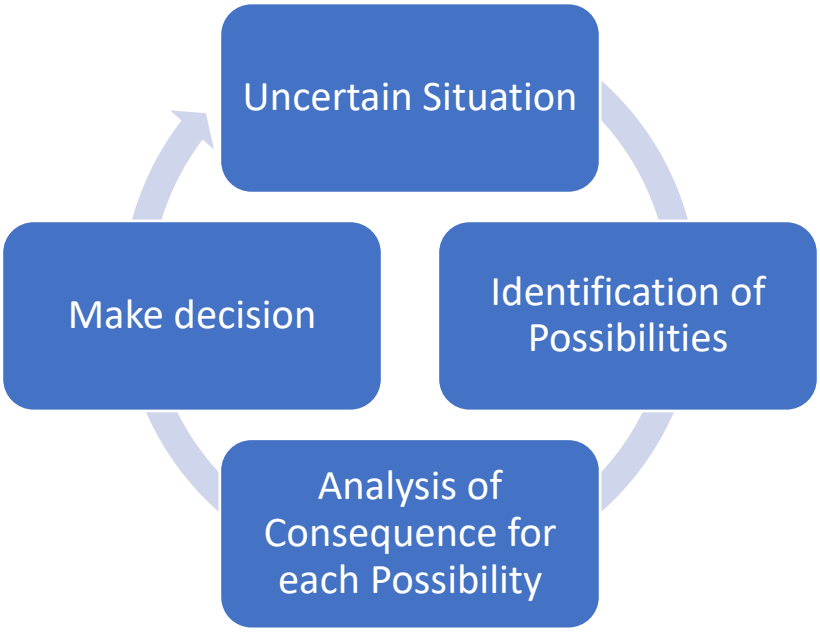

Figure 1. Probabilistic Thinking Model

In this study, probabilistic thinking involves identifying all possible outcomes, assessing the degree of possibility in each case, its consequences, and choosing the best possible way. The decision makers tend to examine the

consequences of their decision to maximize profit (Loewenstern & Lerner, 2003). According to Milkman, Chugh and Bazerman (2009) people's error in assessment of possibilities or consequences can lead to undersaving for retirement, marrying of wrong partners, and acceptance of wrong jobs. Teaching and learning has been largely influenced by the social and cultural contexts (Hod, Bielaczyc, & Ben-Zvi, 2018).

Social and Cultural Contexts

Vygotsky (1978) explored social interaction in the construction of knowledge. The author achieved much success in contributing to the development of knowledge in teaching for conceptual understanding. This theory demand that teachers and learners interact freely in the learning environment. Vygotsky posited that learning should be a product of social, cultural, and political influences. This learning can take place at school, home and workplace. Probability learning does not occur only in explicitly probabilistic situations in classroom (McMullen, Verschaffel & Hannula-Sormunen, 2020). This learning requires teachers to place more emphasis on social interactions, language, experience, and collaborative learning for students' mastery of mathematical concepts. Dayal and Sharma (2020) argued that probability has strong connections with culture. Therefore, understanding peoples' use of probability within cultural context is crucial for teaching and learning. Amir and Williams (1999) study in England on some selected cultures spells out that language, beliefs and experiences had a significant effect on children's probabilistic thinking. Language plays an important role in communication, builds understanding and processes ideas of mathematical concept (Kaplan, Rogness, Fisher, 2014). This idea can be applied in teaching and learning probability when students' experiences about random events such as rolling of a die and tossing of a coin can be explored. People may predict and experiment with the outcomes of dice and coins. People are most likely to predict outcomes of their previous experiences. Then the teacher with much knowledge in probability would have to use critical questions to challenge learners' thinking and reasoning. As questions and answers are used, then the interaction is taking place together with language. How learners express themselves allow the teacher to assess the decision-making of the learners. Game activities provide learners with the opportunity to work in pairs or groups and this is where collaboration is key. In this study, the researcher and the participants engage in non-digital based games.

Probabilistic and Statistical Reasoning for Life

Bilgin, Bulger and Fung (2020) argue that people rely on statistical and probabilistic interpretations for decision making. To support their argument, they cite instances where people, governments, and organizations must depend on statistics for issuing guidelines to prevent the spread of COVID-19. However, a study conducted by Muniz-Rodriguez, Rodriguez-Muniz, and Alsina (2020) on statistical and probabilistic knowledge of Spanish citizens during the COVID-19 crisis, found that citizens lacked good knowledge of statistics and probability to make good decisions. What are the skills and ideas that the curriculum expects learners to acquire in statistics and probability? Do the teaching and learning tasks focus on these skills and ideas? Do the questions asked by the teachers assess these skills and ideas? Why is it important to assess these skills? Assessing these skills would help to determine whether the way of teaching statistical and probabilistic concepts need modification or not for learners' to be well prepared for decision making in this 21st century (Garfield, 1995). Videnovic (2017) found that written

examinations alone are not enough in the assessment of students' conceptual understanding and the call for oral assessment of mathematical knowledge in teaching. The questioning in the form of cross-examination provide guidance and direction for students' learning since there is an opportunity to follow-up with questions as compared to written type questions. Iannone and Simpson (2012) examined advantages of oral assessment compared to other common assessment (e.g., written type) and stated that oral assessment may provide rich information about student skills acquisition. A cross-examination approach gives teachers the opportunity to assess students' probabilistic skills. How are tasks designed to equip students to develop good decision-making skills? National Council of Teachers of Mathematics [NCTM], (1999) refers to probability as "conceived with choices that are based on the likelihood of an event. In real life, making a decision often involves selecting the best possible choice based on the facts available" (p.253). Do teaching and learning provide learners with the opportunity to make choices and to challenge their decisions? This is important in preparing students for real life. When we study probability, it forms the basis for which we decide. This decision may not be perfect since uncertainty is involved, but it may be reasonable. Agus and colleagues (2019) conducted a cross-national study between Italy and Spain on probabilistic reasoning of Psychology undergraduates in verbal-numerical and graphical-pictorial formats and found that Spanish students had more positive attitudes toward statistics, higher confidence and lower statistical anxiety than Italian students. The probabilistic deficiencies leading to bad decisions can be traced to the origin of probability.

Events that Led to Probability Theory

The fundamental principles of probability were formulated by Pierre de Fermat and Blaise Pascal in an exchange of letters between them in the year 1654 (Batanero et al, 2016) while responding to questions posed by a French nobleman who was interested in a game of chance. The problem was to decide whether to bet even money on the occurrence of at least one "double six" in throwing a pair of dice 24 times. Mere's calculation contradicts a well-established gambling rule that indicates a profit for betting on a double six in 24 throws. Right from the onset people tend to live under the mistaken belief that every random situation has a 50-50 chance of occurrence (Gürbüz, Birgin & Çatlıoğlu, 2012; Gauvrit & Morasnyi, 2014). Mere's calculation marked the beginning of the search for probability theory. Developing a mathematical theory of probability that has a precise definition of probability was very difficult at the time. According to a frequentist philosophy of probability, the principles formulated are substantial results in uncertain situations and are the real issues of the field (Franklin, 2016). There were philosophical interests in earlier ideas because it has been difficult to understand uncertain evidence and basic concepts of probability in modern society. "Certain philosophical distinctions are needed in order to identify what body of ideas in ancient and early modern texts should be regarded as probability" (Franklin, 2016, p. 1). The word 'probability' was derived from the verb 'probe' means 'find out' that which is not easily accessible. So, any assessment approach that probe people's thinking and reasoning about uncertainties would provide greater insights on their understanding of probability ideas. This is where cross-examination approach is crucial.

To understand and make meaning of uncertain situations, students should have better probabilistic reasoning by learning the concepts of probability through real-life activities in the form of games. This is because decision-making is highly influenced by the level of one's knowledge. "A goal of statistics education is to enable students' reason about data in the context under conditions of uncertainty and to discriminate between statistical and

probabilistic reasoning and mathematical reasoning" (Batanero & Borovcnik, 2016, p. 3). This is consistent with Engel (2017) who writes that statistics educators have responsibility to ensure probabilistic and statistical literacy society. Martignon and Laskey (2019) intimated that the quality of life of citizens depend on their understanding of risks and ability to make sound choices in uncertain situations. Nonetheless, Rodriguez-Alveal et al. (2022) found that both preservice mathematics teachers and in-service mathematics teachers lack probabilistic thinking skills and to promote the same among students.

Research on Probabilistic Reasoning

Blanco and Chamberlin (2019) indicated that probabilistic misconceptions limit students' problem-solving abilities, and this would affect their ability to make a judgment about uncertainties. Jones et al (1999) conducted qualitative study on 4 students and revealed that overcoming a misconception in sample space, applying both part-part and part-whole reasoning are key in promoting probabilistic thinking among students. Prominent statistics researchers like Batanero and Borovcnik (2016) argued on the uniqueness of statistics and probability in the school curriculum noting that it requires special attention on how concepts are presented to students. These concepts are better presented when teachers are well prepared to teach the concepts with strong evidence of real-life situations for students to reason through and make choices. Based on this, Sharma (2016) proposed activity sequences to aid teachers in addressing the misconceptions of their learners. Nacarato and Grando (2014) developed tasks to investigate 10-12 years old students' probabilistic language and thinking in social practice. Their research found the notions about frequency, chances, possibility and probability are intuitive and how others are mistaken. Based on this, Nacarato and Grando call for teaching approaches to confront and overcome subjectivist probabilistic thinking bias which was found in students' ideas. This call is crucial to statistics teachers, teacher educators and researchers concerned with application of probability in decision-making. However, their study failed to address the question of how such teaching approaches can be organized to address subjectivist probabilistic thinking in students' ideas which may limit their ability to make good decision. It must be noted that the subjectivist probabilistic thinking is not wrong in general. Its validity depend on the context and the knowledge or expertise of the personality involves (Batanero et al, 2016). For instance, an expert mathematics teacher with knowledge in particular examination and a candidate can predict the performance of the candidate in mathematics with a degree of possibility.

Many statistics education scholars (Hokor et al., 2022; Gürbüz, Birgin and Çatlıoğlu, 2012) have examined students' probability knowledge with two dice problems. For example, Hokor et al's study investigate equiprobability misconception using the sum of two dice problem among preservice elementary teachers before college stochastic. Majority of their participants (70.72%) wrongly answered that the sum of 11 and the sum of 10 are equally likely to occur. For instance, when two fair dice are tossed the probability of obtaining the sum of 11 is twice as much as the sum of 12 (Gauvrit & Morasnyi, 2014). However, many people tend to believe that they are both likely to be equal. All these studies rely on written examination in school context which do not provide opportunity to probe students' reasoning or decision with follow-up questions. The majority of the justifications offered by the participants in written cases do not offer greater insights of students' choice of answers (Konold, 1993).

Current research findings suggest that any teaching approach that aims at addressing probabilistic misconceptions improves students' performance better than if the same method is used without targeting misconceptions. One report on constructivist classrooms reveals that preservice teachers were able to confront their misconceptions and deal with them when critical questions were asked to aid them to reflect on their thinking (Hokor, 2020). Some of these misconceptions include equiprobability bias and representativeness bias. In another study, Hokor and Sedofia (2021) examined the probabilistic reasoning of two groups concerning learner-centered approach and teacher-centered approach of teaching targeted at probability misconceptions (equiprobability bias, representativeness bias, belief bias, outcome orientation bias, positive and negative recency effects) and found that preservice teachers' probabilistic reasoning has improved significantly in each group. However, the learner-centered approach was found to be substantially significant compared to the teacher-centered approach in the development of preservice teachers' reasoning on the uncertainties.

The literature reviewed here suggests that while addressing students' misconceptions, professionals' use of probability in real-life context must also be duly considered. In conclusion, Hokor et al. (2022) and Gauvrit and Morasnyi (2014) articles achieve a great deal in using sum of dice problems to identify equiprobability bias within school context but their arguments are limited in out of school context. It also lacks the link between the lessons learned and decision making in life.

Rationale and Purpose

Probabilistic literacy is crucial in the life of adults since they make decisions not only for themselves but also for their children in uncertain situations (Hokor, 2020). Many scientific works and various professionals depend on probabilistic and statistical skills (Sowey, 2020). However, it lacks the needed attention. While there are studies on students' and mathematics teachers' understanding of probability (Bryant & Nunes, 2012; Batanero et al, 2016; Guven et al., 2021), it appears studies that explore non-mathematics teachers' reasoning about uncertainties is lacking. The fact that a concept has been taught doesn't imply its correct usage. Statistics teachers should be aware of professionals' probability knowledge and its use in decision-making (Guven et al., 2021). According to English and Watson (2016) both children and adults share some misconceptions about random events including representativeness bias. Similarly, both children and adults have equiprobability misconceptions (Gauvrit & Morasnyi, 2014). Hokor et al. (2022) investigation identified equiprobability bias, representativeness bias, positive and negative recency effects that impede preservice elementary teachers' ability to solve probability problems. While majority of these misconceptions are found by other statistics education researchers (e.g Gauvrit & Morasnyi, 2014; Ang & Shahrill, 2014), addressing the 'probability misconceptions'in children is problematic in light of other research findings. Sharma (2016) found that several students do not assign reasons for their decisions in uncertain situations making the teaching of probability difficult. This position is consist with Amir and Williams (1999) that children sometimes make correct judgment in uncertain situation but could not provide justifications. Children's first learning involve adults around them. So, adults' reasoning about uncertain situations is most likely to influence children's thinking. Therefore, understanding the reason behind adults' decision-making about uncertainties would provide the basis for structuring teaching and learning for students' first

encounter with probability in classrooms. Sriraman and Chernoff (2020) contend that "a field interested in the teaching and learning of probability and statistics and probabilistic and statistical thinking, such as the fields of probability and statistics education, is uniquely positioned to continue to investigate probabilistic and statistical thinking" (p. 676). I agree and argue that investigating various professionals on the use of probability skills acquired in schools for decision-making would provide broader insights into peoples' probabilistic thinking. Assessing how people use probability skills in decision-making would inform the training of teachers. Estrada and Batanero (2020) claimed that "teachers should also be acquainted with research results that describe children's reasoning and beliefs in uncertain situations for correct intuitions on probability" (p. 3), and I contend that the same argument is sustainable on professional non-mathematics teachers who studied probability previously. Probabilistic and statistical reasoning is important for every profession (e.g psychologists, doctors, journalists and science writers, political analysts, and teachers) (Garfield, 2002). To sum up, understanding people's use of probabilistic ideas daily is necessary if we want to address misconceptions (Sharma, 2016) and prepare students to make sound choices in life. Therefore, this research was initiated to provide an empirical assessment of the use of probabilistic reasoning skills by non-mathematics teachers to make everyday decisions about uncertain situations. The following research questions guided the study.

1. How do non-mathematics teachers think and reason about uncertain situations?
2. How can cross-examination approach be used to promote non-mathematics teacher's probabilistic thinking for making sound choices in uncertain situations?

As much is not known about how non-mathematics teachers' think probabilistically in their daily activities, this study provides insight into how probabilistic skills are used in daily life in making decisions. This study would guide teachers on how to structure their lessons to cultivate sound probabilistic reasoning among their learners in relation to background knowledge. The study contributes to the literature in the development of tasks and critical questions in support of teachers in teaching and learning probability.

Statistics Education

Statistics and probability are part of mathematics which are compulsory in school curriculum from primary to secondary schools (Ministry of Education, 2019; Gürbüz, Birgin & Çatlıoğlu, 2012). The curriculum recommends an inquiry-based instruction. Students are to acquire skills, to analyze and to solve problems both in school and out of school contexts (Ministry of Education, 2010). Also, develop decision-making ability, logical and critical thinking and the ability to reflect critically upon their work and the work of others (Ministry of Education, 2019). However, it appears there is no study that examined people's application of probability skills in everyday life in Ghanaian context.

Also, many of those who teach statistics and probability units do not receive special training on statistics and probability education in Ghana to develop critical thinking and decision-making. Fernández et al (2020) contend that teachers who studied statistics from mathematical perspectives do not have the needed skills for teaching probabilistic and statistical concepts appropriately. Therefore, for teachers to teach probability and statistics effectively, they would need to engage in professional learning by reading research materials on pedagogical knowledge, content knowledge and students' prior knowledge on uncertainties or randomness.

Method

This study used a qualitative design of exploratory research to explore the reasoning of people in an informal educational context in making decisions about uncertainties, after they have studied probability in high and primary schools for several years (Ang & Shahrill, 2014). Qualitative researchers tap into opinions, perceptions, and feelings to provide in-depth understanding of the issue at hand (Stockemer, 2019). Qualitative research employed interviewing and observation of participants as a means to collect data. This type of research make inquiry into the way people interpret and make sense of phenomena of interest (Stockemer, 2019). In this study, a cross-examination approach was used to solicit information from the participants and analyzed to describe the treads about responses to questions on probabilistic thinking tasks. Data collection was done in two weeks.

Participants

The research was conducted in Hohoe, Ghana. The researcher visited two colleges and one public senior high school in Hohoe to discuss his research plans with non-mathematics teachers for their involvement in the study. These institutions were considered because of their proximity to the researcher. These institutions were visited two times each. In the first visit to the three institutions, the researcher met 17 teachers in the 'staff room' excluding mathematics teachers during his two hours stay. Three days later, the second visit was made. The researcher met 11 teachers excluding those he met on his first visit. Out of 28 non-mathematics teachers from the three institutions, only nine agreed to take part in the study and gave their contact numbers for that purpose. Among the nine, three were females and the rest were males. Three of the people who agreed to take part in the study withdrawn at the beginning when contacted on telephone about which day they can be visited for the interview with reasons that they were busy.

The rest six (two females and four males) took part in the study but only four males agreed for their data to be used. The participants in this study were conveniently sampled because they were the only people who were willing to take part in the study. Etikan, Musa and Alkassim (2016) intimated that convenience sampling is a nonprobability sampling technique that a researcher uses to choose a sample of subjects or participants from a population. This type of sampling has a lot of limitations due to how the participants are involved in the study. Even though it may be argued that this type of sampling does not ensure representativeness of the population under study, nevertheless, "it is useful especially when randomization is impossible like when the population is very large" (Etikan, Musa & Alkassim, 2016, p. 1). It can be used when the researcher does not seek to generalize the entire population (Etikan, Musa & Alkassim, 2016). One out of four participants was a first degree holder and the rest of them were master's degree holders. Their average years after probability studies is 14.5-years. Two of the participants had science educational backgrounds. One with physical education and the other, psychology background. One of the participants teaches in high school and the rest teach at the College of Education level where teachers are trained to teach at primary schools.

The researcher solicited information from the participants about their profession and some social games they engaged in during their leisure time before a date was set for the main interview. One major question used to

solicit information from people before the design of the tasks was what game they liked playing during their leisure time. When they mentioned a game, then they were asked to describe how the game is normally played. Some of the games mentioned by the participants were rolling of dice, playing cards and spinning.

The tasks focussed on dice because it was the most game played by all participants (both males and females). One of the task was on insurance because half of the participants owned a car. According to Zimmer, Schade and Gründl (2008) people buy insurance to protect themselves in the events of accident which might involve small probability. The participants claimed even though they engage in the games for fun, they want to win. According to the participants, some people engaged in it for financial rewards, but they do not. This information serves as a prerequisite for designing the tasks. Three tasks were designed. All the four participants responded to two tasks on dice, but only two responded to the task on insurance. This was because only two participants had a car and that was insured. These tasks test the participants' decision-making ability in situations of uncertainty. The tasks are presented next.

The Sum of Dice Problem. Two people decide to play a game with two dice. There are two options to choose from. These are "the sum less than or equal to 6" and "the sum greater than 6". Each person holds a die and puts it into the bowl. The bowl is then covered. The two dice in the bowl are then shaken five times by a referee. After which the bowl would be opened. When the sum is '6 or less', A wins then B lose. When one wins, a point of 1 is recorded and zero for the other. After that the bowl would be covered and shaken again. The game is played 25 times. The number of times the game has to be played should be odd and reasonable to reflect the reality of the game. The odd number of times makes it easy to determine the winner. The one with the highest points wins.

Repeated Die Problem: A die is to be tossed five times. In the first place, one would have to toss the die four consecutive times and outcomes would be written. Secondly, one would have to predict the next likely outcome(s) in the fifth toss to win. This game was played individually and observed by the researcher.

Insurance Problem: Two insurance companies have the following policies for same type of car and people with similar driving experiences on the roads. The roads are identical except in distance. Insurance company M: Anyone who drives 100km (from town A to town B) would have to pay Gh¢120 and 200km (from town C to town D) would have to pay Gh¢120 as a premium for a year. Another insurance company P offered the following for the same car on the same two types of roads. Insurance company P, anyone who drives 100km (from town A to town B) and 200km (from town C to town D) would have to pay Gh¢100 and Gh¢140 respectively as a premium per annum. Note that you can only insure with only one company. Also, you can insure against one/two road(s) with the same company. The purpose of this task/problem is to examine peoples' ability to identify what is reasonable or fair by considering the risks involved.

Interview: Cross-examination Approach

An interview in the form of cross-examination was used to collect data on how the participants reason to decide on which choice they made on uncertainty tasks. This form of assessment provides detailed insights

about participants thinking and reasoning in uncertain situations (Theobold, 2021). In a cross-examination approach, tasks were designed based on the experiences of the participants and questions were asked directly to test their decision-making ability. This approach appears natural than written assessment. The cross-examination approach involves interrogating the decision of the participant in uncertain situation. This form of assessment differs significantly from written type because of its dialogic nature (Iannone, Czichowsky, & Ruf, 2020), and how it assesses the uniqueness of each participant's experience since the follow-up questions may vary from participant to participant. The purpose here was for the participant to reflect on their thinking and reasoning that led to the choices they made. The questions were intended to help the participants establish the truth or identify the best possible choice under the circumstance.

Participants' responses to questions form the bases for the subsequent questions. Oral assessment provides the opportunity to probe peoples' thinking compared to written counterpart (Theobold, 2021) by providing in-depth understanding of the participants' probabilistic thinking. Videnovic (2017) opined that oral examinations are effective for conceptual understanding while written examinations are appropriate for procedural understanding. The oral questions may be both open-ended and close-ended. This approach would enable the participant to consider various alternatives before making decisions. This approach required fast critical questioning skills and good knowledge on the area under examination since one would have to tap on the participant's responses. Based on this, it may be difficult for some teachers to implement.

The participants were able to explain their thinking and reasoning that led to their choices. The interview was recorded and transcribed for analysis with the permission of the participants. The participants were interviewed between 30 to 60-minutes on different days. The interview took place at the apartment of each participant. The interviewee has the right to withdrawn from the study at any time and are not compelled to answer any question.

Data Analysis

As the study seeks to examine non-mathematics teachers' thinking and reasoning about uncertain situations, it has qualitative characteristics (Ozmen & Baki, 2021). Qualitative research make use of interpretive approach to reality and life experience of people (Stockemer, 2019). Therefore, analysis was done qualitatively.

Content analysis was carried out on the recorded interview between the researcher and the participants to interpret their probabilistic thinking. The recording was transcribed in their entirety (Linell, 1994) by description: word-for-word, speech-neutral text, organized in dialogic form. The participants' responses to the tasks were presented and analyzed.

The participants' responses to each task were examined to identify their reasoning ability. This helped to know whether the participants considered various possibilities and its consequence concerning a particular uncertain situation. The researcher used this to conclude whether or not the decision made by the participants was the most reasonable decision under the circumstance.

Results

The research questions for this study are (1) *how do non-mathematics teachers think and reason about uncertain situations? and (2) how can cross-examination approach be used to promote non-mathematics teachers' probabilistic thinking for making sound choices in uncertain situations?* To answer these questions I carefully examined four professionals non-mathematics teachers' reasoning about uncertain situations.

How People Think and Reason in Uncertain Situations

Data were collected on the participants' reasoning through cross-examination about uncertain situations on daily activities or games they engaged in. The interest here is to know whether the participants apply probability skills in decision-making correctly and to test the impact of cross-examination on their thinking and reasoning ability.

Participants' Response on Two Dice Problem

Interview with Peter

> *Researcher: Two dice are to be tossed, 'the sum less than or equal to 6' or 'the sum greater than 6' which one do you prefer if you want to win?*
>
> *Peter: The sum less than or equal to 6.*
>
> *Researcher: Why?*
>
> *Peter: I'm sure I can win with my preferred choice.*
>
> *Researcher: Why not the sum greater than 6?*
>
> *Peter: Whether 'a sum less than or equal to 6' or 'a sum greater than 6' all give the same chance of winning. A sum greater than 6 is difficult to win with. I always win with a sum less than or equal to 6.*

This is an indication that Peter's prediction was based on a previous encounter with the sum of dice game. Peoples' mistaken beliefs may be because people might not play this game continuously for several times to ascertain the reality.

> *Researcher: Do you think if we play this game 25 times you can win?*
>
> *Peter: Yes, I will win. So researcher chose the sum greater than 6. The game was then played 25 times under supervision of a chosen referee of Peter.*

At the end of the game, the researcher recorded 14 representing 56% wins against Peter's wins of 11 representing 44%. The following were the recorded scores.

> Researcher:10100011111010101100110 01
>
> Peter: 01011100000101010011001 10

Therefore, the researcher was judged the winner of the entire game. The researcher then asked Peter if he knew that whoever chose the sum greater than 6 has more chance of winning than whoever chose the sum less than or equal to 6. Peter replied, "that is not true because if you take each die the numbers are from 1 to 6. And half of 6 is 3. So two dice will give the highest sum to be 12 and half of 12 is 6. You are just lucky today and not anything else. Peter's reasoning about numbers on a die is correct. That is true for a die when one chooses (1, 2, and 3) and another chooses (4, 5, and 6), they all have the same chance of winning. But same can not be applied to the sum of two dice.

Researcher: Let us check all the sums of two dice. Both find all the sums of two dice as 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

This is where Peter realized that 7 is the median for the sum of numbers on two dice and not 6 or 6 and 7 as compared to 3 and 4 as the middle numbers on a die. Peter further argued that "even though 6 is not the median, it does not necessarily mean we don't have an equal chance of winning." Based on his argument we listed all the sums together because he had difficuty doing it alone as shown below. He then counted them.

Dice	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Researcher: What have you noticed?

Peter: I am really surprised they don't give an equal chance of winning. I haven't noticed this before.

This shows that Peter has been complacent in analyzing the possibility before prediction. Also, an indication that his decision was based on habit.

Researcher: What lesson did you learn from this game for life?

Peter: The lesson I learned is that I need to investigate issues rather than taking decisions based on assumptions. Initially, I thought they would give the same chance of winning.

Peter does consider the outcomes in his initial reasoning, but assumes that the two options are equally likely. He is recognizing the need to consider the likelihood of the outcomes, but he does not assess these likelihoods accurately in the first instance. Peter even draws on his school knowledge of the probabilities of the outcomes from a single dice in his reasoning. Peter's demonstration of sound reasoning by the decision he made after cross-examination shows how cross-examination approach can be used to promote probabilistic thinking among students.

Interview with Rafael

Researcher: 'The sum less than or equal to 6' or 'the sum greater than 6' which one do you prefer?

Rafael : I prefer the sum less than or equal to 6.

Researcher: Why?

Rafael: The possibility of getting a sum higher than 6 is difficult.

Researcher: You think you can win if we play this game 25 times?

Rafael: Yes, I will win.

The game was played between the researcher and Rafael. Below are the sequences of outcomes.

Researcher:111001111110110110011110

Rafael: 000110000001000100110000 1

The researcher got 18 scores representing 72% and Rafael had 7 scores which represent 28%. This again affirms

the fact that the possible ways of obtaining a sum greater than 6 is more than obtaining the sum less or equal to 6.

Researcher: What did you think caused your loss?

Rafael: I have not done much calculation initially.

Researcher: Which calculation?

Rafael: I should possibly consider all the possible sums.

Researcher: So, given the second chance which one would you chose?

Rafael: I would choose the sum greater than 6.

Researcher: why?

Rafael: I just feel that this would lead to a win.

Researcher: So, what about your calculation?

Rafael: I can't do any calculation about this. I can hardly remember how to go about it.

The researcher showed the chart of the sum of two dice to him to examine carefully. Rafael, oh ok, I see. This response from Rafael indicates that his second decision to go for a sum greater than 6 was only based on the fact that he lost the game on his first choice of a sum less or equal to 6.

Researcher: What have you noticed?

Rafael: I noticed the sum greater than six are more.

Researcher: What lesson did you learn from this game for life?

Rafael: In life what you expect may not come true. In deciding what to do in any situation which has more than one option to select from, I would need to assess the strength of each case before I choose.

Rafael draw upon previous experiences in his initial decision making, which is often what is required in real-life probabilistic situations where the probabilities of specific outcomes are not known, or even the possible outcomes are not known. He tend to apply probabilistic thinking skills in real-life generally, rather than applying the specific probabilistic thinking skills that are applicable to situations with known outcomes where the probabilities can be quantified.

Interview with Selorm

Researcher: 'The sum less than or equal to 6' or 'the sum greater than 6' which one gives a higher chance of winning?

Selorm: We all have an equal chance.

Researcher: Why?

Selorm: Because the outcomes can either result in low or high.

The low here, refers to a sum less than or equal to 6 while the high means the sum greater than 6.

Researcher: Is this game fair?

Selorm: Yes, it is fair.

Researcher: Why did you say it is fair?

Selorm: It is fair because when you toss the two dice the sum may either be less or equal to six or the sum may be more than six.

The response suggests that every random event has an equal chance of occurring.

Researcher: Two dice are to be tossed, 'the sum less than or equal to 6' or 'the sum greater than 6'

which one do you prefer?

Selorm: I prefer a sum less than or to equal to six.

Researcher: Why not the sum greater than 6?

Selorm: That is the one I expect can easily give me a win.

The game was then played. Below were the sequence of outcomes.

Selorm:001010100110010100010001

Researcher:110101011001101011101110

The total scores for Selorm and Researcher are respectively 9 (36%) and 16 (64%). This mean the sum greater than 6 is more probable.

Researcher: Is there a reason why you lost the game?

Selorm: Because of the way die was shaken. If it is well shaken, it gives a sum less than or equal to 6.

Researcher: Okay, then shake it well. The following were the results obtained out of his twelve rounds of shaking the bowl well:

Researcher: 101100110110

Selorm: 010011001001

At this point, Selorm reframed the initial position to suggest that "a die gives a higher number than a low number". Selorm's reasoning was now based on what happened between him and the researcher, but not on the analysis of the number of possible sums in each case. Critical questions are important to help people confront their thinking.

Researcher: Study this chart carefully and compare the 'sum less than or equal to 6' to the sum greater than 6.

Selorm: The sum of the numbers on the two dice greater than 6 is more than the sum less than or equal to six after counting.

Researcher: What lesson did you learn from this game for life?

Selorm: I need to study things carefully before I make a choice. One needs to understand the consequence of decisions before you make them in life.

The response revealed that the predictions were not done based on the analysis of possible outcomes.

Selorm just like Peter draw upon previous experiences in the initial decision made, which is normally needed in real-life uncertain situations where the probabilities of possible outcomes are not known, or specific outcome is not known. Selorm demonstrated general knowledge of probability but not specific to this context as the question required. However, the departure from general probabilistic thinking to specific context shows how cross-examination approach can be used to promote sound probabilistic thinking in people.

Interview with Joseph

Researcher: What are the numbers on the die?

Joseph: We have 1, 2, 3, 4, 5, and 6.

Researcher: Two dice are to be tossed, 'the sum less than or equal to 6' or 'the sum greater than 6' which one do you prefer?

Joseph: I do not have much knowledge about probability. It was my difficult concepts in high school. So I can't answer questions about it.

Researcher: This is just a game that you said you have been playing.

Joseph: Yes, I have been playing it. I even have a die here look at it. But the question you asked involved probability. To answer that question you need knowledge on probability.

Researcher: But you studied probability in high school.

Joseph: So one has to list all the sums on the dice before one chooses.

Researcher: Ok, then list the sum. The researcher noticed he was not able to list all the sum, but then asked, "which one do you think will be more?"

Joseph: It is fair as it relates to probability because no one can manipulate the outcomes. At first, I thought the sum greater than 6 are more possible but after second thought they would all be the same.

The researcher showed a chart of sums of numbers on the two dice to him to study carefully.

Researcher: Do you still hold the view that it's a fair game?

Joseph: I will go for a sum greater than 6.

Researcher: Why?

Joseph: It is easy to win because it has more possible outcomes.

One reason for Joseph's inability to make the right choice initially was due to failure to consider all possibilities. So when all possible outcomes were put before him, he was able to demonstrate good probabilistic thinking.

Researcher: What lesson did you learn from this game for life?

Joseph: Analysing all possibilities helps you to make a fair judgment about an event or things. Also, it helps avoid bias in terms of decision making.

Joseph's initial response is an interesting one as he immediately identifies it as a school-type question, rather than a real-life question. The difficulties Joseph appears to have stem from identifying the sample space (as described by Bryant and Nunes), rather than the other aspects of probabilistic thinking.

Participants' Reasoning on Repeated Die Problem

On tossing a die several times, all the participants demonstrated the same level of reasoning. For example, all the participants agreed that when a die is a tossed once any number on the die may show up. However, when each participant tossed a die four times and outcomes are written, they are asked which number is most likely to show up in the next toss then they refer to the numbers that had not occurred yet. But, all the numbers on the die are equally likely. The thinking of the participants here shows that the teaching of probability can be difficult. However, teachers can use a series of activities to challenge every assertion of their learners.

Participants' Response on Insurance Problem

Interview with Joseph

Researcher: Which of the insurance companies do you think is fair?

Joseph: All the insurance companies are fair because they tried to reduce the amount to get the needed customers. If you find the average amount for company P it is the same as the first one. It is just a business strategy.

Researcher: If you have an insurance company which pricing type would you adopt, and why?

Joseph: I don't see the difference between the two companies.

Researcher: But if Evans insures his car with company P on a 100km road he would have to pay Gh¢100

compared to Mawutiem who agreed with company M and drives on the same 100km road would have to pay Gh¢120.

Joseph: The difference in the amount is insignificant.

Researcher: What do you think influences the insurance company's premium rate or the amount charged?

Joseph: I don't think there is anything that influences their premium. However, I think profit influences the premium. Credibility too does. They want to make name for themselves. Also, for sustainability as well.

Researcher: Do you think the distance one drives increases the possibility of one getting involved an accident.

Joseph: No, an accident does not depend on the distance covered.

Researcher: Don't you think the statistics on the road could be considered by the insurance company before arriving at the premium rate?

Joseph: I think they will consider it.

Researcher: What about the driver getting tired.

Joseph: That one cause an accident.

Researcher: If tiredness leads to an accident, then can't we say distance has an influence?

Joseph: Yes, we can. The driver can easily lose control when tired and this could result in an accident.

Researcher: The driver who drives on a 200km road and the one who drives on an 100km road, which one is more likely to get tired?

Joseph: The one who drives on 200km road.

Researcher: So, do you think charging the same price for the two distances is fair?

Joseph: In this case, it is not fair. But they want to make profit.

Researcher: Which other case do you think would be fair?

Joseph: I think having different premiums for each road. That would be ok.

Researcher: Then don't you think the insurance company P has been fair.

Joseph: "Yeah, I think so, I think so," he said repeatedly.

This affirmation by Joseph has once again proven the importance of critical questioning on students' conceptual understanding. Cross-examination is crucial in promoting sound thinking among people and this would lead to good decision-making.

Interview with Peter

Researcher: For insurance company P, anyone who drive 100km (from town A to town B) and 200km (from town C to town D) would have to pay Gh¢100 and Gh¢140 respectively as a premium per annum. Which of the insurance companies do you think is fair?

Peter: All of them are fair.

Researcher: Do you think the distance one covers affects accidents?

Peter: Yes, it has an effect. As you are tired, your concentration level is low as compared to driving short distances. Also, it can lead to speeding which can cause an accident because of temptation to reach your destination quickly.

Researcher: If it affects an accident, then which one will you prefer?

Peter: I prefer company P because of its pricing.

Researcher: But you said all the companies in terms of pricing are fair, can you explain further? Peter: They are all fair because the total for all is the same.

Researcher: Which of the pricing types would you prefer if you have an insurance company?

Peter: I would prefer P because many people would like it. It is a good marketing strategy. Researcher: Why did you prefer the P strategy?

Peter: Short distance would come with a low accident as compared to long distance. The risk involved in driving on a 100km road is lower.

Cross-examination promotes good probabilistic thinking. The reasoning exhibited by Peter, he should conclude that company P is fairer.

Researcher: Do you think all the companies are fair based on risk?

Peter: They are all fair.

Even though Peter's response revealed that short distance would lead to low accident, he was unable to admit that company P was fairer. However, Peter prefers company P to company M which is a good decision. The choice of Peter suggests the importance of cross-examination in people's decision-making. The next section presents the discussion under themes.

Discussion

This study investigated professional non-mathematics teachers' probabilistic thinking in making decisions in uncertain situations through cross-examination.

Probabilistic Thinking in Uncertain Situation

The initial responses of all the participants suggest that every uncertain situation implied equally likely (Hokor et al. 2022) as such they did not consider all the possibilities before deciding on which options to choose. Similarly, they also held a belief that the least sums were more likely compared to the higher sums. However, the sum less than or equal to 6 is 15 out of 36 (41.67%) while the sum greater than 6 is 21 out of 36 (58.33%). This means the higher sum is more likely. Additionally, they had difficulty identifying all the possible outcomes and this also affected their decision-making ability. However, with critical questions, they were able to overcome their difficulties and biases.

Questions that require students to offer explanations or justifications to their predictions and to reconcile their results with predicted outcomes promote good probabilistic thinking among the participants. Mueller, Yankelewitz and Mahner (2014) found that questions play an important part in students' mathematical achievement. This is an indication that when teachers planned their lessons well with critical questions being asked timely, students would be able to identify all possibilities, analyze the possibility of each occurring and its consequences before a decision is made. This would enable students to respond to the changing needs of society. Table 1 presents lessons learned by the participants after responding to questions on two dice tasks.

Table 1. Presents Summary of Lesson learned on Uncertainty Task

Participant	Lessons from the Sum of Two Dice Problem
Peter	I need to investigate issues rather than taking decisions based on assumptions. Initially, I thought they would give the same chance of winning.
Rafael	In life what you expect may not come true. In deciding what to do in any situation which has more than one option to select from, I would need to assess the strength of each case before I choose.
Selorm	I need to study things carefully before I make choice. One needs to understand the consequence of decisions before you make them in life.
Joseph	Analyzing all possibilities helps you to make a fair judgment about an event or things. Also, it helps avoid biases in terms of decision-making.

The responses from the Table 1 suggest that when tasks or activities are well planned with critical questions, students would tend to be critical of everything before making choices. And this is crucial for this century which is full of information (Engel, 2017). Tasks must be designed to allow students to provide justifications for their solutions or predictions (Nacarato & Grando, 2014). The remarks by the participants on the lessons learned from the game affirm the results of Mohd and Shahrill (2021) that a non-digital game-based learning approach enhances students' understanding of mathematical concepts. Game-based lessons should be encouraged in probability classrooms. But the emphasis should be on the probability ideas and skills behind it, not fun. Even though the fun aspect should not be completely ignored. This would promote sound probabilistic thinking in students to avoid people taking decision on the basis of emotions or habits.

Probabilistic Thinking and Reasoning on Repeated Die

The responses on "repeated die problem" suggest that the die has a memory and natural balance (Hokor, 2020). People easily tend to depart from their belief in the least opportunity. They are of the strong view that the die would result in one of the numbers that have not occurred yet compared to those that had occurred. This view by the particiapants indicates that peoples' immediate experience has a significant effect on their decision-making ability. This finding affirms the view of Dale (2015) that people tend to premise their judgment towards more recent information for which the latest news suffers from that base. I agree with the author and argue that facts can also be distorted or supported by current results. So, teaching activities should be well planned with critical questions to support students' prior sound knowledge on uncertainties. Additionally, teaching and learning should be designed to challenge their prior experiences or misconceptions and this would equip them with sound experiences for life choices.

Making Decision on Fairness

On insurance problem, the participants with good reasoning would know that the longer a policyholder drives, the higher the chance of getting involved in an accident and filing a claim. One would therefore see company P's premium policy as better than company M's. Initially, both participants concluded that accidents do not have any

relationships with distance or the more policyholder drives. However, with critical questions, both agreed that the more one drives the higher the possibility of getting tired and that can lead to an accident. While Joseph reverses his decision to say company P is fairer compared to company M, Peter still maintains the initial position that none of the companies was fair in terms of the premium policy. His reason for this position was that on average both companies were charging the same amount for the two roads. However, Peter prefers company P to M. This study has once again emphasized the importance of probing and pushing questions in mathematics lessons (Titin, 2014). The real-world knowledge of students needs to be understood because it serves as a basis for instruction. Teachers' knowledge on this informal knowledge of students is in line with constructivist view and are useful for making instructional decision (Zieffler, 2008). Cross-examination approach has the potential to increase learners' self-efficacy (Theobold, 2021) because learners would develop sound probabilistic thinking. This approach may be great in evaluating understanding of probabilistic ideas and developing oral communication skills in students (Iannone & Simpson, 2012). The reasoning of the participants on this task was not sound leading to irrational decision (Konold et al., 1993). In training teachers to teach probability, developing questioning skills among them is of special interest because of the unique nature of probability.

In summary, the participants of this study had difficulty making good choices in situations of uncertainty. This is because their knowledge of uncertainties was limited. This finding is in support of Muniz-Rodriguez, Rodriguez-Muniz and Alsina (2020) who claimed that there are deficiencies in citizens' statistical and probabilistic knowledge which may result in wrong choices with serious consequences for the entire population. Good questioning can help people to overcome their biases in uncertain situations. The questions should challenge students' thinking or results. It appears the teaching of probability is challenging (Ben-Zvi & Makar, 2016; Estrada & Batanero, 2020) because what people believe in is sometimes difficult to disprove practically (Hokor, 2020). For example, some students believe that the outcomes of coins and dice are determined by God (Amir & Williams, 1999). Also, some students believe that when they predict outcomes; they will surely come to pass, else they are only not lucky in that case. Since in most cases they do not assign reasons for their luck, it is difficult to provide them with counterexamples or reasons for them to confront (Sharma, 2016). However, if adults and children share similar biases (Gauvrit & Morasnyi, 2014), and adults in most cases give reasons for their choices or decisions as seen in this study, then it can serve as a basis for designing teaching and learning activities for children's first encounter with probability in classrooms.

Conclusion

I have argued that teachers of probability and statistics as part of their content knowledge need to know how probability skills acquired in schools are used in decision-making about uncertain situations. For pedagogical knowledge teachers should know how to use cross-examination approach to assess or develop probabilistic thinking of their students for making best choices in uncertain situations. The participants of this study tended to make judgments out of habit. They appear to be complacent and failed to consider all possibilities before concluding. The consequences of each possibility have been overlooked. Due to this, they were not able to make good choices. Bad choices are inevitable if one has probabilistic biases which cause one to fail to examine the risks or the consequences of each possibility (Dale, 2015). Therefore, it is important to design tasks that allow

students to thoroughly explore the solutions about various uncertain situations in both formal (classroom-based) and informal (out-of-classroom) contexts through games. When teachers cultivate the habit of exploring probabilistic situations with students, they would tend to be critical about anything that requires decision-making. The paper calls for reflection on the current way of teaching probability to integrate probabilistic skills into tasks consciously. The evidence from both tasks and questions suggest that a mixture of questions somewhat like a cross-examination can help people to overcome their mistaken belief that every random event has equal chance to occur. Cross-examination approach should be at the heart of teaching probabilistic concepts to provoke students' thinking and reasoning. The analyses of the results suggest that the cross-examination approach as an assessment tool of probabilistic reasoning provides more insights. This study advocates for 'cross-examination approach' for development of probabilistic thinking. This would help accelerate probability skills in students for decision-making. The future application of this cross-examination approach, extend to statistical literacy and mathematical thinking, and might be useful in predicting how students could use probabilistic and statistical reasoning skills in uncertain situations. Even though cross-examination has been found to be effective to cause people to reflect on their decision, it may not be good for all students with various learning styles (Theobold, 2021). Furthermore, some teachers may need more time to acquaint themselves with questioning skills to provide more insights of students' probabilistic thinking. Similarly, this approach may be difficult to implement in classroom situations where we have many students. In this case, designing a common game based on prior experiences of all learners and exploring their reasoning could be difficult. Nonetheless, learners can work in groups based on their experiences or games they played. Finally, probabilistic thinking within the framework of statistics education requires conscious plan effort to develop.

Contribution to the Literature

The major contribution of this paper is the link between probabilistic thinking in uncertainties and the study of statistics education for real life and new insights it provides into the use of cross-examination approach to provoke peoples' probabilistic reasoning development for decision-making. The study places importance in understanding professionals' probabilistic skills usage for inform choices to provide direction for instruction. This paper also adds to the literature transfer of probability skills acquired in schools and its uses in real-life through cross-examination.

This paper uncovers a potential for strengthening the status of probabilistic thinking in statistics education through tasks and real-life game using cross-examination approach. This study makes considerable progress towards integrating real-life tasks or non-digital game-based learning that contributes to the development of probabilistic thinking for decision-making. Drawing on peoples' daily experience to designed real-life non-digital based-game learning and cross-examination approach of "probabilistic thinking", the current research establishes point of reference to develop probabilistic thinking among students to make sound decision in uncertain situations daily.
The implications related to the improvement of the assessment of probabilistic thinking. Cross-examination provides more accurate investigation of probabilistic thinking and reasoning with respect to the use of game-based problems. Furthermore, the effort to construct problems based on daily experiences of people and examined the same provides true picture of professionals' probabilistic literacy. This research contributes to effective and

efficient teaching of probability concepts relating to tasks and questioning to prepare students for this dynamic world.

Limitation

This study also had limitations because only those who are willing were involved in the study. The analysis focuses on the knowledge that the four non-mathematics teachers had on probability and its uses in making decision. Therefore, findings may not reflect the entire population. Also, this study was limited in terms of few numbers of participants; therefore additional studies involving more professional groups are needed to shed more light on the issue under study. Same tasks or new tasks may also be used in this direction. Additionally, the tasks in this study focused on few games and daily activities. Therefore, any researcher interested in this area should expand the tasks to cover more games to provide more insights on people's thinking and reasoning in uncertain situations. Also, it is possible the reader of this work may have different interpretation about the tasks on insurance as this problem reflect distinct characters of the probability, where exact solution is not always the case as expected in mathematics.

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