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Abstract

The primary author interviewed participants individually and conducted a focus group interview with three preservice mathematics teachers (PSMTs) attending a southern university in this interpretive case study. The authors also gathered the preservice teachers' final reflection papers related to a mathematical letter writing exchange (MLWE) in which they participated. The two research questions guiding this study were: 1. What were the PSMTs' perceptions of value after participating in a MLWE with high school students? 2. How did PSMTs' thinking about the qualities of a good mathematical task evolve as they participated in a MLWE? For this study, the primary author used open coding and inductively analyzed the interviews. In comparison, directed content analysis guided analysis of the reflection papers to determine the PSMTs' perceived value of the mathematical letter writing exchange experience. Findings suggested that PSMTs should form strong relationships with students to help students become more confident in doing mathematics and, as a result, be more successful in fully engaging with mathematics. Also, explicit and implicit structures existed in the descriptions of a good mathematical task. For example, one intrinsic theme indicated that PSMTs should consistently prioritize mathematics in their feedback and discussions with students.

Introduction

Here is a short passage from Alvin's reflection paper (obtained January 14, 2020):

The biggest thing I learned from the letter writing experience is that it is worth it to take the time to get to know students so that you can find ways to make the learning more engaging. I have always believed that student-teacher relationships are important for learning, but this project proved just how important it was.

Also provided is a brief excerpt from Simon's individual interview transcript (conducted October 9, 2019):

Researcher: *What do you think you might have done differently? ... Or what do you think you might do differently in the next two mathematical tasks that you've not done in the first three?*

Simon: *We haven't made stories in the first three, and we want to work on that. I want to make it as fun as possible then. And we want to work on that... Yeah. Yeah. With the stories and stuff.*

Researcher: *I love that. What do you think you will put in the stories to make it as fun as possible?*

Simon: *“Once upon a time, there was a dragon.” Like, he got into basketball and stuff like that. Like the story behind the picture. And then, a story about the task. Like, the grid and the dragon is [sic] right here, and he moves out here. So, the dragon’s traveling down half-court, and he’s trying to take a shot at the three-point line.*

In the short opening quote, Alvin indicated what she had learned from participating in the mathematical letter writing exchange (MLWE). In the excerpt from the transcription, Simon gave insight into her perspective of a *good* mathematical task. This study investigates the value that preservice mathematics teachers (PSMTs) attributed to their participation in a MLWE with high school students and the evolution of how they described what a *good* mathematical task was.

In response to the Common Core State Standards (CCSS), posing mathematical tasks has become a more integral part of instructional practices because they lead students to a deeper understanding of mathematics (Letwinsky & Cavender, 2018). All mathematics teachers should include mathematical tasks in their classroom pedagogy (Anderson & Signe, 2011). Yet, Crespo (2003) mentioned that posing mathematical tasks is a challenge for preservice teachers learning how to teach mathematics. Participation in a MLWE allows preservice teachers access to real-world experiences of posing practical mathematical tasks and giving students feedback while having the benefit of time to reflect on their teaching practices (Crespo, 2002).

Six years ago, the primary researcher read Anderson et al. (2009/2010) and started the MLWE between PSMTs at a regional university and high school mathematics students. In contrast, Anderson et al. (2009/2010) focused mainly on the logistics of a MLWE and the Algebra II teachers’ and high school students’ reactions. They did not focus on the PSMT’s reactions to the MLWE. This study investigated PSMTs’ perceptions concerning a *good* mathematical task and the value the PSMTs attributed to their participation in a MLWE with a class of rural geometry students.

Mathematical Tasks

Mathematical problems, written as a task, are meant to engage students and help students identify real-world problems that employ the math skills these students have learned in class (NCTM, 2009). As such, Izci and Caliskan (2017) suggested that posing appropriate open-ended mathematical problems [e.g., mathematical tasks] should take a more critical role in aiding students’ understanding of mathematical content. The authors also indicated that if PSMTs could experience developing successful open-ended mathematical problems for students, they might see their efforts as a valuable tool to promote students learning of mathematics.

According to Henningsen and Stein (1997), three aspects of a *good* mathematical task fall under two dimensions of developing mathematical tasks. They are features of the task and cognitive demand. Mathematical tasks should feature multiple entry points, several ways to represent a solution path, and possibly more than one correct answer. Cognitive demand is related to the types of thinking the student must use to solve the task. One

might categorize cognitive demand into various levels of thinking. For example, the student may follow a predetermined number of steps (generally a lower level of cognition) to arrive at an accurate answer. **Or** the student may rely on their knowledge of mathematics to synthesize their strategy for solving the task (generally a higher level of cognition).

Mathematical tasks are an excellent way to access students' learning since they "produce qualitative and quantitative data [that]... ensures fairness of measurement and supports robust learning" (Izci & Caliskan, 2017, p. 472). For this study, the PSMTs reflected and improved upon developing mathematical tasks based on student feedback throughout the MLWE. Based on the participants' shared experiences in developing and creating mathematical tasks, this research adds to the literature by including the PSMTs shared collective of attributes they used to describe a *good* mathematical task.

History of PSMTs Participating in a MLWE

Prior studies indicated that PSMTs benefit from participating in a MLWE. Findings from Fennel's (1991) narrative reports and surveys indicated that the preservice teachers seemed to share positive reflections for the MLWE in general. At the end of participating in a MLWE, the PSMTs seemed to understand the mathematical content better. They also shared that they could better understand their penpals' interests, attitudes, and learning progression.

In a second study, the preservice teachers seemed to gain insight into crucial mathematical communication strategies. Additionally, the PSMTs debunked some preconceived notions about student achievement related to student interest in mathematics. These skills are vital tools these PSMTs will regularly use in their future mathematics classrooms (Phillips & Crespo, 1996).

A third study reported that PSMTs seemed

to "move beyond superficial considerations to a deeper[,] more critical analysis" (Sally) and become "more insightful" (Terry); and allowed them "in time to (estimate) guess a little better why students answered questions in a certain way" (Megan). Writing and reflecting on students' work with an explicit focus and format played an important role in developing preservice teachers' interpretations. (Crespo, 2000, p. 175)

If these PSMTs had not participated in this MLWE, they would not have had these rich and enlightening work experiences that helped them think about their students' responses more deeply.

In these past studies, PSMTs reported a deeper understanding of the content, how students learned mathematics, a better means of discussing mathematics, and delivering helpful feedback. Also, PSMTs realized that all students could learn mathematics despite their initial impression of their penpals' abilities and motivations. Finally, Crespo (2002) reported that a MLWE allowed PSMTs time to focus on student responses. Without the delay of dropping off and picking up letters regularly, PSMTs might not have had the time to reflect on their penpal's learning progression.

This study hopes to add to the literature surrounding MLWEs by investigating the value participants reported while participating in this MLWE. Above and beyond this, the majority of MLWEs occurred with elementary and middle schools. In contrast, this study investigated data collected from a MLWE between high school students and two middle school PSMTs and one secondary PSMT.

Setting

This study was conducted with PSMTs at a university located in a rural region in the Southeastern U.S. During the 2017–2018 school year, IES:NCES (n.d.) reported that the students attending this university were White, 84%; Black, 4%; Hispanic, 3%; Asian, 2%; mixed races, 3%; and other, 4%. They also reported a similar proportion of male (54%) to female (46%) students. All preservice middle and secondary mathematics teachers pursuing an undergraduate- or graduate-level degree in mathematics education at this university are required to take three mathematics methods courses.

The MLWE conducted over the past six years had around 12 to 15 preservice teachers enrolled each semester in one of those methods courses. There were 14 PSMTs enrolled in the mathematics methods course in the semester (Fall 2019), in which the researcher collected data for this study. During the past six years, all of the PSMTs were White. During Fall 2019, this continued to be the case. Therefore, this class seemed to parallel prior course enrollment and school demographics. Furthermore, Motoko (2014) reported that “more than three-quarters of all teachers in kindergarten through high school are women... up from about two-thirds three decades ago” (para. 3). This pattern extended to the mathematics methods course at this university. In Fall 2019, there were 12 female and two male PSMTs.

Research Statement

In this interpretive case study, the primary author interviewed individually and in a focus group two middle and one secondary PSMTs attending a southern university and collected and analyzed mathematical tasks and letters exchanged by participants and their penpals at a rural high school to understand teachers’ perceptions of posing *good* geometry mathematical tasks and their perceptions of value after participating in a MLWE.

Research Questions

This study addressed two questions using an interpretive approach: How does PSMTs thinking about the qualities of a *good* mathematical task evolve due to participating in a MLWE? What value do PSMT’s attribute to participating in a MLWE with high school students?

Methodological Framework

For this qualitative study, the authors chose a framework that employs an interpretative perspective. Studies related to posing mathematical tasks and mathematical letter writing exchanges used interpretivism as their

theoretical perspective (Anderson & Signe, 2011; Crespo, 2000, 2002, 2003; Fennel, 1991; Henningsen & Stein, 1997; Phillips & Crespo, 1996). Maxwell (2013) stated researchers are interested in physical and behavioral events that take place and “how the participants in your study make sense of these, and how their understanding influences their behavior. This focus on meaning is central to what is known as the ‘interpretive’ approach to social science” (p. 30).

This research emulates studies by Anderson and Signe (2011), Crespo (2000, 2002, 2003), and Phillips and Crespo (1996), who also used a case study approach in their studies. Therefore, a case study methodology appeared to be an appropriate choice for this study. Creswell (2007) defined a case study as a type of methodology that “is a qualitative approach in which the investigator explores a bounded system... over time, through detailed, in-depth data collection involving multiple sources of information..., and reports a case description and case-based themes” (p. 73). The bounded system in this proposal included all middle and high school PSMTs enrolled in the mathematics methods course at a southern university in the fall of 2019.

Theoretical Framework

Maxwell (2013) argued that “what people perceive and believe is shaped by their assumptions and prior experiences” (p. 43). Chilisa and Kawulich (2012) stated that constructivists believe that “truth lies within the human experience” (p. 10) and “that knowledge is subjective” (p. 10). Since each human experience is different, multiple truths may exist without “any hint of a critical spirit” (Crotty, 2003, p. 58). Additionally, the “understanding of this world is inevitably our construction, rather than a purely objective perception of reality, and no such construction can claim absolute truth” (Maxwell, 2013, p. 43).

In regards to letter writing, Crespo (2000) described how mathematical letter writing “provided a context that resembled the interactive nature of teaching practice, [sic] but without the immediacy and pressures for action that characterize[s] actual mathematics classrooms” (pp. 157–158). In this study, middle and secondary preservice mathematics teachers constructed their knowledge regarding mathematical tasks while they developed mathematical tasks for a MLWE during one semester. Therefore, this study employed a constructivist stance to inform the research further.

MLWE Logistics

The primary researcher compiled data from participants’ letters and mathematical tasks, individual and group interviews, and reflective papers. Furthermore, the primary researcher used inductive analysis and content analysis to review the interview data and reflection papers. The trustworthiness of findings was established in this study by (1) keeping a reflexive journal, (2) member checking, (3) peer review, and (4) triangulation of data through interviews, reflection papers, letters, and tasks.

Patton (2015) described purposeful random sampling as a means by which “random selection will avoid controversy about potential selection bias” (p. 268). The primary researcher was purposeful in selecting a

specific group of preservice teachers. However, the three participants who volunteered for this study were left to chance and are, in effect, random.

The high school geometry students initiated the letter exchange by writing a letter of introduction to a preservice teacher. The introduction included general information about the high schoolers' interests inside and outside school-related coursework and activities, the students' perceived mathematical abilities, and the individuals' attitudes toward mathematics. The PSMTs responded with an introduction letter and enclosed a grade-appropriate, standards-based mathematical task (see Table 1).

Both preservice teachers and the geometry students maintained anonymity by using self-selected pseudonyms. Letters and mathematical tasks were alternatively picked up and delivered so that each class received a response on a biweekly basis. Each class received six letters each throughout the letter exchange, and the preservice teachers sent five mathematical tasks during this semester. Furthermore, the PSMTs sent tasks aligned with the high school students' geometry standards, as indicated by Table 1.

Table 1. MLWE Logistics

Item	Student → Preservice Teacher		Student ← Preservice Teacher		
Letters	Letter 1	8/27/2019	Letter 2	9/3/2019	
	Letter 3	9/10/2019	Letter 4	9/17/2019	
	Letter 5	9/24/2019	Letter 6	10/01/2019	
	Letter 7	10/08/2019	Letter 8	10/22/2019 ¹	
	Letter 9	10/29/2019	Letter 10	11/05/2019	
	Letter 11	11/12/2019	Letter 12	11/22/2019 ²	
	Completed Tasks (CT)	Geometry Standards	Feedback on Task (FB)		
Tasks	CT 1	9/10/2019	G.CO.A.1-5	FB 1	9/17/2019
	CT 2	9/24/2019	G.CO.A.1-5	FB 2	10/01/019
	CT 3	10/08/2019	G.CO.A.1-5	FB 3	10/22/2019 ¹
	CT 4	10/29/2019	CO.A.1-B.8	FB 4	11/05/2019
	CT 5	11/12/2019	CO.A.1-C.11	FB 5	11/22/2019 ²

Note: ¹ The high school's fall break created an expected delay on 10/15/2019. ² Snow days created an unexpected delay on 11/19/2019.

PSMTs and Their Penpals

Table 2 gives a brief description of the PSMTs. All of the participants were female. All three of them were at different levels of their education as well. Two of the participants, Simon and Theodore, were pursuing their undergraduate degrees in secondary mathematics and multidisciplinary studies (middle school mathematics). Furthermore, Alvin had transitioned from her undergraduate program (elementary education) to pursuing her master's degree in STEM education.

Table 2. Participant Demographics, Teaching Experience, and Penpals

	Alvin	Simon	Theodore
Gender	Female	Female	Female
Education Level	Masters in STEM Education	Secondary Education Mathematics	Multidisciplinary Studies
Previous Aspiration Education Background	Chemical Engineer Chemical Engineering (two years), Elementary Education	Third Grade Teacher	Nurse
Preservice Teacher Teaching Experience	Summer Camp Elementary Students, Practicum Elementary and Middle schools	Practicum Middle and High Schools	Tutoring Middle and High School Students, Substitute Teacher Middle School, Job Shadow Middle School
Penpals	Dramagirl Shared: Baseballfan, Footballfanatic	SoftballQueen (moved away), Dragonfyre Shared: Mr. Popular	Baseballfan, Footballfanatic Shared: Dramagirl

Note: All PSMT and penpal names are pseudonyms.

Additionally, each applicant brought different experiences in formal education and nonformal education practices. Although, all three individuals had not worked as professional teachers in each of their respective fields. Alvin and Theodore were partners in the mathematics methods course, and Alvin partnered with a PSMT who did not participate in this study.

Each participant had a different experience with their penpals. Alvin was able to write to the same penpal during the entire letter exchange. Theodore wrote to two students throughout the program, and Simon had a penpal that moved away about midway through the letter writing exchange. The mathematics methods instructor and I assigned Simon another penpal she could write to for the program's remainder. All three participants reported that even though they did not have direct contact with their PSMT partner's penpal(s), they thought of them as their penpals as well.

Table 3 provides brief descriptions of the geometry penpals. Since most high school credits assign geometry in the 10th grade, the high school students were all 10th-graders except for one. Dramagirl had just moved to the high school from out of state and had not taken geometry yet. The high school class had 12 male and six female

students enrolled, and the same ratio of high school students participated in this study. Mathematical ability is based on the students' prior standardized test scores and GPA.

Table 3. Penpal Information

Student Pseudonym	Grade	Gender	Mathematical Ability
Dramagirl	11 th	Female	Average
Baseballfan	10 th	Male	High
Footballfanatic	10 th	Male	Gifted
SoftballQueen	10 th	Female	Average
Dragonfyre	10 th	Male	High
Mr. Popular	10 th	Male	Average

Findings

PSMT's Perception of Value after Participating in a MLWE with High School Students

Feelings

Although all three participants had never heard of a MLWE, they were excited to participate in this assignment, and they continued to enjoy the assignment throughout the correspondence. Theodore reported, "This is an assignment that, in a way, did not feel like an assignment." Simon stated, "When I first heard about the project, I was mostly [*sic*] excited about writing letters to students." Finally, Alvin said she thought it was "a cool project. I enjoyed doing it. I think it's a neat experience." These results emulated the positive perspectives preservice teachers shared in the study by Fennel (1991).

"I Learned."

Simon stated, "My feelings have not changed towards mathematics, but my ideas have changed about how to encourage students to learn and persevere in math." Simon also indicated in her reflection paper that she consistently discussed her penpals' mathematical abilities in the letters and mathematical tasks. She maintained a mathematical discourse to enrich the letters and encouraged her penpals to continue the conversation. Both of her penpals responded positively to this mathematical discourse and confirmed Simon's consistent and systematic way of presenting and prioritizing mathematics in the context of this mathematical exchange. In a way, it appeared that Simon considered the letters as an extension to the mathematical tasks.

Additionally, the three participants described how they had developed their knowledge of giving and receiving feedback. The PSMTs discussed how they constructed mathematical tasks that incorporated their penpals'

interests in unique and creative ways. Moreover, Theodore stated how they had become more familiar with the geometry standards, finding valuable resources, and the importance of developing relationships with their penpals. Furthermore, to create engaging and relevant tasks, Theodore further explained that this experience required “us [to] remember back to high school years when we took geometry.”

Relationships

As Alvin’s short passage at the beginning of the article seemed to portray, relationships were essential. The PSMTs shared several stories about the relationships they had with their teachers while in high school. These experiences shaped the PSMTs’ initial perspective on developing a relationship with their respective penpals. However, the MLWE helped the three participants see the significance of creating and maintaining these relationships.

Furthermore, even though the participants identified how much time and effort were required to develop and sustain these relationships, they also stated how influential these relationships were to student learning. After writing one or two letters, these PSMTs had established secure connections with their penpals. Alvin indicated that her relationship with Dramagirl was why her penpal tried to solve all the mathematical tasks she received, including the Algebra II problem presented within one of the mathematical tasks Alvin and Theodore sent. Even though Dramagirl did not solve the mathematical task correctly, Alvin remained positive and stated that she was “proud of her [penpal] for trying.” Simon wrote in her reflection paper that

as a teacher, I can have such a great influence on my students—even indirectly through letters! My students showed that they really appreciated how my partner and I related the tasks to their interests. Dragonfyre writes, “I like how you use the things I like in the math problems.”

Baseballfan wrote to Theodore, “Thank you for the support as well. I’m not dissing my past teachers, but they never give me compliments after usually [*sic*] like the sixth week, because they already expect the kids to get the answers correct.”

“I Love Math.”

Not surprisingly, the PSMTs reported that they loved mathematics, although for different reasons. Alvin loved mathematics because she could see how mathematics was in everything. Simon stated, “I liked math and science growing up. And just the further up I went in the totem pole of math classes in high school, I really just stuck with math because it made more sense.” Moreover, Theodore developed her love of mathematics in high school after she started tutoring in mathematics.

In contrast, all three teachers stated that geometry was their least favorite subject in mathematics. Again, all three participants gave three different reasons for this strong dislike. Alvin alluded to her high school geometry teacher as the reason for not liking geometry. Simon described a lack of confidence in doing geometry proofs

that dissipated when working in a group setting. Theodore stated that she had difficulty remembering geometry and had to go back to her notes to retrieve geometry concepts.

Challenges and Suggestions

The main challenge that all three participants experienced was creating or revising mathematical tasks to fit the geometry standards. Also, “the standards did not change, and this made it difficult for my partner and I to create different tasks that were not repetitive.” This shared sentiment led to a common suggestion proposed by all three participants. They recommended that PSMTs enrolled in future classes of the algebra-based mathematics methods course write to a class of Algebra I or Algebra II students rather than geometry students.

It is notable that when the MLWE began six years ago, the students were writing to algebra students. Since then, the high school teacher’s schedule changed such that she only teaches geometry. This content switch from algebra to geometry has undoubtedly created tension (albeit slight) when asking students in an algebra methods course to correspond with students in a high school geometry course.

Challenges, Strategies, and Suggestions

All participants shared a common challenge addressing the individual penpals’ combined interests in a group mathematical task. In this study, the participants overcame this challenge with creative and unique solutions. Alvin and Theodore indicated how they found out their penpals all shared a common interest in scary movies. Since Halloween was coming up, Theodore stated:

I think that we’re going to try and relate this next one towards like a [*sic*] Halloween-related stuff. Where we haven’t done like kind of like [*sic*] themes in the past like that. We’ve just done to [*sic*] what they liked. So, like one of them mentioned, he liked the movie, *It*. And the other one mentioned she just likes watching scary movies and stuff. And the other one mentioned that he just likes watching is [*sic*] like scary stuff like that. So, we kind of are thinking maybe we’ll go along the lines of something Halloween. And then on the last one, it’ll be right there before Christmas. So, we’re thinking [of] maybe doing something like Christmas or New Year’s or even Thanksgiving. Like relating it to that, but still relating it to their interests as well.

Simon took another route. As prescribed by the short excerpt of the transcript displayed at the beginning of this study, Simon stated that she and her partner would create a storyline for their final tasks. However, as Simon thought ahead to a classroom setting, she indicated that it might be challenging to develop a story that incorporated the interests of a class of 25–30 students. Simon suggested that mathematical tasks should include a component that allows the students to make the mathematics relevant to themselves. For example, she did a classroom activity during her practicum that involved students’ birthdays. Simon’s birthday was on May 6. Simon indicated how her birthday digits: zero, five, zero, and six could create two binomials $(x + 6)$ and $(x + 5)$. Once multiplied, Simon stated that the students “made it relative to themselves—about their personalities.”

Finally, Alvin and Theodore also described how challenging it was to scaffold tasks that negotiated their penpals' different mathematical abilities. They overcame this obstacle by creating several challenge questions for the higher-achieving student, Footballfanatic, in their group. Even though Alvin and Theodore catered to their highest achieving student in the group, which coincided with findings from a study by Crespo (2003), they slightly modified their focus. The two preservice teachers wanted all of their penpals to succeed at doing mathematics even though they had different abilities in mathematics. Even though the preservice teachers overcame the challenges associated with different mathematical abilities, Alvin and Theodore suggested that the high school teacher should group students by their interests and abilities.

Imagine if this had been the case for Alvin and Theodore. These prearranged groups of penpals would have robbed Alvin and Theodore of learning how to develop creative ways to meet their penpals' unique interests while differentiating the tasks to challenge the different ability levels present in their group for this study. Therefore, even though Alvin and Theodore may not have recognized their achievement in overcoming these challenges at this time, they might be confronted with similar challenges in the future and remember back to this experience.

Summary

In this study, all three participants stated that they would like to host a MLWE in their future classrooms. Furthermore, these future mathematical exchanges might be modified to fit the participant's unique teaching styles and classroom needs. For example, Alvin and Theodore want to teach middle school mathematics, so they thought they might partner with a high school mathematics class to conduct their MLWE. These future iterations of MLWEs might be exciting for educators to consider for their classrooms to encourage mathematical communication.

Preservice Teachers' Thinking about the Qualities of a *Good* Mathematical Task evolved due to Participating in a MLWE?

Strategies, Challenges, and Resources

Alvin and Theodore collaborated with each other, created networks with other teachers, and sought their professors' advice. These resources remedied the challenges a lack of resources Alvin and Theodore experienced at the beginning of the MLWE. Alvin stated, "I almost wish that I had like the textbook from the rural class." Theodore described how "very, *very* difficult finding websites and places to get math tasks from" was. Towards the conclusion of the MLWE, all three participants reported using Google to find mathematical tasks aligned to the appropriate standards. Alvin shared that her professor at the southern university directed her to an online resource, *Robert Kaplinsky's* website. The skytyper task Alvin's group sent to their penpals came from this resource. Alvin and Theodore also obtained ideas from the *Illustrative Mathematics* website. Above and beyond this, Alvin, Simon, and Theodore talked about their newly created mathematical tasks and resources with several veteran mathematics teachers.

Feedback

Feedback included two dimensions: feedback directed towards the penpals and feedback directed towards the preservice teachers. Simon stated that feedback should include what the student did well, what the student might improve, and a strategy to help equip the student with future mathematical approaches. Alvin and Theodore appeared to follow this approach, as well. Furthermore, Simon and Theodore also asked questions in their letters to glean feedback and encourage their penpals to respond. However, when their penpals did not respond to their questions, Alvin and Theodore created a short survey to elicit feedback on any thoughts their penpals might have towards the mathematical tasks that coincided with a study by Crespo (2000). Theodore said:

I do think it would be beneficial—not only for us—but also for them to explain, “Why?” Maybe they didn’t get done with a problem or why they had trouble with the problem. And then that way, we know how to adjust and make the next task. That way, if it is too difficult, we can scaffold it and bring it down a notch and get it back to their level. That way, they are still learning, and they aren’t just completely not doing their task because it is too hard.

The survey asked the students to rate the mathematical task’s difficulty level and how much they liked it. A third question remained open-ended for the penpals to write what they did not understand about the task and what they liked or disliked about the task.

Finally, Simon stated, “In the letters, my student would let me know their mathematical abilities and how they felt about the task, but my partner’s student would hardly mention the task.” As a result of the MLWE, Simon also described how her feelings had changed since she could hear direct feedback from her penpals concerning their mathematics perceptions.

My student, Dragonfyre, would mention that he thinks math is “very useful.” I found that the encouragement I write in the letters truly makes a difference in the students’ perceptions of their abilities to do math. My other student, SoftballQueen, wrote, “Thank you for having confidence in me with math. It really helps me try harder.”

Similar to the study conducted by Anderson and Signe (2011), Simon acknowledged her penpal’s feedback as significant and built her mathematical tasks and responsive feedback in a way that helped her students feel more empowered to do mathematics.

Definitions and Characteristics

When the researcher asked each participant what they initially thought a mathematical task was, the participants’ consensus indicated that a mathematical task was “like a problem or a set of problems that you have to like solve.” However, as the semester went on, Alvin’s definition changed to reflect how a mathematical task could be a thematic set of mathematical problems that progressively lead students to think more deeply about mathematics.

Simon indicated that a mathematical task could be an activity, and Alvin said it could be a project. Though Theodore agreed with these ideals, she challenged these concepts by stating that the activity or project needed to fit a worksheet format. She also indicated that each mathematical task should start with the activity, project, central question, and pertinent questions.

Above and beyond this, the three participants characterized mathematical tasks as meeting each penpal's unique interests and ability levels. Simon came up with a creative story about a dragon playing basketball to review rigid motions with her penpals. Alvin and Theodore focused on including challenge questions that met the varying mathematical abilities of their penpals. These findings contradicted Crespo (2000), who determined that preservice teachers seemed to lower their expectations for lower-performing students and cater to the higher ability students. Simon's description of Mr. Popular and Dragonfyre addressed a possible reason for this inconsistency.

Whereas the other guy [Mr. Popular]. He's not so much interested. He also just kind of probably does his work. And like, "Hey, help!" kind of thing. When he can actually do it, he's maybe just be [sic] like, "Why do it when you can have somebody else show you?" I do, like, really try to understand the students. But—so, yeah. I think they're—they can both do the work. I think the sporty guy can do the work. If he just saw that it was useful to him. Dragonfyre, he's pretty good. He's very accurate and precise in his drawings, and he has to be for a guy who draws dragons. So, I think that they're both capable.

Summary

According to the PSMTs, mathematical tasks seemed to have explicit and implicit structures that made the tasks *good*. Even though a *good* mathematical task's intrinsic characteristics were more challenging to uncover, these structures were concepts that the PSMTs considered incorporating into their future teaching practices. The participants crafted their mathematical discussions so that their students felt safe to enter and sustain a mathematical conversation without fear of any negative repercussions. Simon appeared to be better at this since she prioritized mathematical communication in her letters and her mathematical tasks. Additionally, Alvin, Simon, and Theodore avoided adverse language and tried to incorporate a positive perspective in their feedback to their penpals, even when the students were wrong. This positivity helped students change their perspective of their mathematical abilities for the better.

Conclusions

Before the MLWE experiences, the PSMTs shared how critical developing relationships with their students and giving feedback was in their future teaching practices. However, after participating in this exchange, they could see why these teaching aspects might be essential to encounters with their future students. Their positively worded feedback and close relationships encouraged their penpals to try harder on the mathematical tasks because they perceived that the PSMTs cared about them and their mathematics learning.

Furthermore, the PSMTs identified the importance of aligned geometry tasks that combined students' interests and were differentiated to account for student ability. To create or find mathematical tasks that met their expectations, the preservice teachers networked with seasoned teachers, professors at the university, and found multiple websites that helped them find *good* mathematical tasks. Finally, these resources may be helpful to these PSMTs when they start teaching full-time.

Above and beyond this, the PSMTs discovered that feedback is not a one-way discussion. For the PSMTs to deliver *good* mathematical tasks, they needed their penpals' opinions of their mathematical abilities, hobbies/interests, and perception of the sent mathematical tasks. This feedback helped the PSMTs develop or find mathematical tasks that engaged and empowered their penpals in doing mathematics and showed how the PSMTs listened and cared about their penpals' opinions.

Implications for Future Research

Alvin, Simon, and Theodore acknowledged that geometry was their least favorite mathematics subject. Even though Alvin stated that she had a gap in her mathematics education from Algebra II, she still identified geometry and statistics as her least favorite mathematics subjects. Simon mentioned how she wished she had gained more confidence in her earlier grades so that she might have "stronger confidence in higher proofs." Theodore only said, "I am not a huge fan of geometry."

Even though all three participants shared an appreciation and love for mathematics, they also shared an aversion to geometry. Since this study focused on PSMT's perception of a MLWE, the author did not probe the participants further about their dislike for geometry. However, future studies might further investigate PSMT's perceptions and feelings associated with geometry.


References

- Anderson, N., Rutledge, Z., Hall, K., & Norton, R. (2009/2010, December/January). Mathematical letter writing. *Mathematics Teacher*, 103(5), 340–346. <http://www.nctm.org/publications/>
- Anderson, N., & Signe, K. (2011). Learning to pose cognitively demanding tasks through letter writing. *Journal of Mathematics Teacher Education*, 15(2), 109–130. <https://doi.org/10.1007/s10857-011-9193-9>
- Chilisa, B., & Kawulich, B. (2012). Selecting a research approach: Paradigm, methodology and methods. In C. Wagner, B. Kawulich, & M. Garner (Eds.), *Doing Social Research: A Global Context* (pp. 51–61). McGraw Hill.
- Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education*, 3(2), 155–181.
- Crespo, S. (2002). Praising and correcting: Prospective teachers investigate their teacherly talk. *Teaching and Teacher Education*, 18(6), 739–758. [https://doi.org/10.1016/S0742-051X\(02\)00031-8](https://doi.org/10.1016/S0742-051X(02)00031-8)
- Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics*, 52(3), 243–270.

- Creswell, J. W. (2007). *Qualitative inquiry & research design: Choosing among five approaches* (2nd ed.). SAGE.
- Crotty, M. (2003). *The foundations of social research: Meaning and perspective in the research process* (2nd ed.). SAGE.
- Fennel, F. (1991). Diagnostic teaching, writing and mathematics. *Focus on Learning Problems in Mathematics*, 13(3), 39–50. <https://www.questia.com/library/p408/focus-on-learning-problems-in-mathematics>
- Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*. 28(5), 524–529. <https://doi.org/10.2307/749690>
- Illustrative Mathematics. (2011). *High school geometry*. <http://tasks.illustrativemathematics.org/HSG>
- Izci, K. & Caliskan, G. (2017). Development of prospective teachers' conceptions of assessment and choices of assessment tasks. *International Journal of Research in Education and Science (IJRES)*, 3(2), 464–474.
- Kaplinsky, R. (2014, February 11). *How do skytypers write messages?* <https://robertkaplinsky.com/work/skytypers/>
- Letwinsky, K. M. & Cavender, M. (2018). Shifting preservice teachers' beliefs and understandings to support pedagogical change in mathematics. *International Journal of Research in Education and Science (IJRES)*, 4(1), 106–120. <https://doi.org/10.21890/ijres.382939>
- Maxwell, J. A. (2013). *Qualitative research design: An interactive approach* (3rd ed.). SAGE.
- Motoko, R. (2014, September 6). Why don't more men go into teaching? *New York Times*. <https://www.nytimes.com/2014/09/07/sunday-review/why-dont-more-men-go-into-teaching.html>
- National Council of Teachers of Mathematics (NCTM), (2009). *Focus in high school mathematics: reasoning and sense making* (3rd ed.).
- Patton, M. Q. (2015). *Qualitative research and evaluation methods* (4th ed.). SAGE.
- Phillips, E., & Crespo, S. (1996, February). Developing written communication in mathematics. For the Learning of Mathematics, 16(1), 15–22. <https://flm-journal.org/Articles/86C0E4183701E637B4243BF22CF36.pdf>
- U.S. Department of Education. Institute of Education Sciences National Center for Education Statistics (IES:NCES). (n.d.). *Percent of all students enrolled, by race/ethnicity: Fall 2018* [Chart]. <https://nces.ed.gov/ipeds/use-the-data>

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