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Developing Probabilistic Reasoning in Preservice Teachers: Comparing the Learner-Centered and Teacher-Centered Approaches of Teaching

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Abstract

Many studies have examined the learner-centered and teacher-centered approaches of teaching, but none seems to have compared the two approaches in developing preservice teachers' probabilistic reasoning by targeting probability misconceptions. This paper calls for assessment of the two teaching methods based on recent findings and new methodologies. This study, therefore, examined preservice teachers' probabilistic reasoning by comparing the learner-centered and teacher-centered approaches of teaching probability targeted at addressing probabilistic misconceptions. In this quasi-experimental research, 59 preservice teachers comprising 32 in Group A and 27 in Group B were used. The results showed that learner-centered approach had a statistically significant positive effect on preservice teachers' probabilistic reasoning [F (1, 56) = 114.955; p = .000 < .05]. This means that the learner-centered approach was more effective than the teacher-centered approach in developing preservice teachers' probabilistic reasoning. Thus, it was recommended that the learner-centered approach of teaching directed at addressing misconceptions be used to develop probabilistic reasoning in preservice teachers to help improve their problem-solving ability in probability.

Keywords: Probabilistic reasoning, Constructivism, Teacher-centered approach, Misconceptions, Preservice teachers, College of education

Introduction

An essential goal of teacher education is to prepare preservice teachers to become effective classroom teachers who foster student learning (Santagata & Sandholtz, 2019). In order for teachers to teach Mathematics more effectively, their knowledge of Mathematics and the skills of teaching the subject are very crucial. Effective teachers are an important precondition for student learning (Aina, Olanipekun & Garuba, 2015). Effective teachers require three kinds of teacher knowledge: subject matter knowledge, pedagogical content knowledge, and curriculum knowledge (Shulman, 1986). Subject matter knowledge is the knowledge of the subject without consideration for the way of teaching the subject. Pedagogical Content Knowledge (PCK) refers to how students learn; knowledge about teaching approaches and different theories and assessment procedures. Pedagogical content knowledge is of special interest because it identifies the distinctive bodies of knowledge for teaching.

Curriculum knowledge, on the other hand, is the teachers' understanding of the curriculum. Kleickmann et al (2013) found that teachers' content knowledge is a reflection of their pedagogical content knowledge. Bolyard and Packenham 2008; and Sanders and Morris 2000 (cited in Cantoy, 2010) observed that even though content knowledge is vital, several research findings suggest effective mathematics teaching depends on adequate teacher pedagogical content knowledge.

Research suggests that students' learning of mathematics is greatly influenced by the method of instruction adopted by the teacher (Sharma, 2016). Mathematics can be taught using the learner-centered approach or the teacher-centered approach. A popular learner-centered approach is constructivism. Constructivism is a learning theory which states that the learner is responsible for creating new knowledge for themselves and that new learning is influenced by past knowledge and experiences (Baerveldt, 2013). In a constructivist classroom, instructional strategies like group work and discussions are employed (Denton, 2012). According to Clements and Battista (1990), learner-centered approach for teaching a specific topic in Mathematics has the following features:

i. *"Knowledge is actively created or invented by the learner, not passively received from the environment.*
ii. *Students create new mathematical knowledge by reflecting on their physical and mental actions.*
iii. *No one true reality exists, only individual interpretations of the world.*
iv. *Learning is a social process in which students grow into the intellectual life of those around them.*
v. *When a teacher demands that students use a set of mathematical methods, the sense-making activity of students is seriously curtailed" (pp. 34 and 35).*

Thus, constructivism states that learning is an active, contextualized process of constructing knowledge based on personal experiences of one's environment rather than acquiring knowledge through rote learning. In this study, therefore, constructivism is conceptualized as helping preservice teachers to construct the ideal meaning of probability under the guidance of a teacher to help preservice teachers improve and develop a conceptual understanding of the concept and for improved teaching outcomes. If preservice teachers are to construct the ideal meaning of mathematical concepts in order to apply and teach them well, they need to be placed at the center of the teaching and learning process. Although the learner-centered approach of teaching mathematics has been found to cause students' performance in mathematics to improve substantially (National Council of Teachers of Mathematics, 2003), it has been observed that some classrooms in Ghana are still characterized by the teacher-centered approach of teaching.

Lim and Hwa (2007) found that teaching and learning of Mathematics in schools is full of the teacher-centered approach of teaching and textbook-oriented method where learners memorize mathematical formulae and laws without understanding the concepts. "It seems that most teachers still prefer the traditional way of delivering explicit mathematics instruction, sometimes combined with challenging questions and mathematical discourse with the students to promote conceptual understanding and critical analysis of the mathematical context" (Voskoglou, 2019, p. 1). The lecture method is a typical teacher-centered approach to teaching. As an approach, lectures are less likely to provide instructors with feedback about student learning and rest on the presumption that all students learn at the same pace. Moreover, students' attention wanes quickly during lectures and information tends to be forgotten quickly as students are mostly passive learners in this kind of instruction. Finally, lectures emphasize learning by listening, which is a disadvantage for students who prefer other learning styles (Schwerdt & Wuppermann, 2011). The lecture method of teaching mathematics may cause students to lose interest in learning mathematics since it makes learners passive rather than active participants in the learning process.

Many countries have made probability an integral part of the school curriculum (Jun, 2000). Although probability forms a critical part of most mathematics curricula in many countries, the training that preservice teachers need to teach probability is missing in some training programs (Batanero & Diaz, 2012). Meanwhile, Roseth, Garfield and Ben-Zvi, (2008) argued that probabilistic reasoning is necessary for students to be informed citizens in the 21st century. In the present study, probabilistic reasoning refers to the way people reason about uncertainty situations and making decisions based on likely outcomes. Bennett and Anway (2003) defined probabilistic reasoning as "the way people reason about likelihood (of outcomes) and with uncertainty" (p.138). Probabilistic reasoning involves:

i. *identifying a desired outcome or a situation that involves uncertainty;*
ii. *using theoretical probability to determine the likelihood of that outcome; and*
iii. *making a decision with a level of confidence estimate directly from the likelihood* (Nguyen, 2015).

To develop better probabilistic reasoning in students, teachers' correct knowledge about probability is needed. When teachers are equipped with appropriate skills, their confidence in teaching probability increases. But some studies have revealed that the current way of training teachers do not provide the essential skills needed to teach probability effectively (Jones, 2005; Khazanov & Prado, 2010). Research has shown that despite the numerous interventions in helping to improve students learning outcomes, individual learning style is very key in our quest for improved teaching and learning of mathematics (Johnson & Dasgupta, 2005). Learning style serves as the reference point for curriculum development and instruction.

Some preservice teachers come to mathematics classrooms with the erroneous view that mathematics is an exact or a fact discipline, and as such everything about it should be memorized. Weldeana and Abraham (2014) revealed that prospective teachers hold traditional beliefs about mathematics, limiting their intellectual development in the discipline. Khazanov & Gourgey (2009) argued that if we expect improvements in students' conceptions, then instructional interventions need to be designed to eliminate students' misconceptions of probability. Additional research is also required to clarify the essential components in the preparation of teachers to teach probability, identify adequate methods, and establish appropriate levels at which each component should be taught (Batanero & Diaz, 2012). Despite the importance of probability and statistics, there is limited research about instructional methods and their effects (Shay, 2008), and it appears few research examined the effect of the learner-centered and teacher-centered approaches of teaching statistics on students' understanding in classrooms (Weltman & Whiteside, 2010; Lessani, Md. Yunus & Bakar, 2017). Lovett and Lee (2017) examined the preparedness of high school preservice mathematics teachers' ability to teach statistics and the results revealed that the cohort of preservice teachers entering high school mathematics classrooms are not prepared to teach statistics.

Some studies have examined the learner-centered and teacher-centered approaches of teaching mathematics on students' problem-solving abilities, but the evidence seems to suggest that none of the studies provides an empirical comparison of the learner-centered and teacher-centered approaches targeted at addressing probabilistic misconceptions in relation to developing the probabilistic reasoning of preservice teachers. Additionally, it appears there is no study that has compared the effect of the learner-centered and teacher-centered approaches of teaching on preservice teachers' probabilistic reasoning at the colleges of education level in Ghana. Therefore, this study was designed to address the above gaps. The important hypothesis that this study sought to test is: There is no significant difference in the probabilistic reasoning (mean scores) between students who experience the learner-centered approach targeted at addressing probabilistic misconceptions and those who experience teacher-centered approach targeted at probabilistic misconceptions in the post-test. It is hoped that this study will contribute to the teacher development literature and provide a strong empirical basis for teaching statistics and probability in colleges of education. In the rest of the study, we present the review of relevant literature and the methodology employed for the study. It is followed by the presentation and discussion of results, conclusions, practical implications, and limitations.

Literature Review

Theoretical Review

This study is anchored on the socio-cultural constructivist theory of Vygotsky (1978) which contends that learning occurs at two levels: first, in the context of shared social activity known as the social or interpersonal plane, and second, on the individual plane. Learning at these two levels can occur in any context, whether during mathematics lessons, in the workplace, or in the home. Vygotsky's work considered reconstruction of knowledge in a social context. According to him, language plays a crucial role in cognitive development because children's knowledge, ideas, and values develop through interactions with others. Vygotsky believed that a learner's acquisition of knowledge takes time. This suggests that teachers should exercise patience for learners to explore concepts in order to acquire all the necessary knowledge.

One key aspect in the socio-cultural theory is the concept of "zone of proximal development" (ZPD) which according to Vygotsky, is the difference between a child's current performance (the problems the child can solve independently without any support) and the level of performance that the child could achieve with guidance from an adult or by working with a more experienced person such as a teacher, a parent, a peer or a sibling. Vygotsky thus advocated that for the learner to learn independently, there should be an experienced person to support the learner. He contends that the learners' intellectual life will be greatly developed when teachers succeeded in leading learners to reorganize their existing ideas with new ones in a meaningful way. This indicates that learners are to be at the center of teaching and learning, interacting with their own colleagues while teachers serve as facilitators. It is crucial that teachers are well prepared to facilitate meaningful learning else learners will be unable to learn effectively. The socio-cultural constructivist theory is relevant for this study because research by Sharma (2016) showed that the theory plays a crucial role in students' ability to confront their own probabilistic misconceptions and resolve them meaningfully.

Teacher-Centered Approach versus Learner-Centered Approach

In this study, the teaching method in which the teacher lectures or stands in front of the class and orally presents his or her teaching note to the learners is called the teacher-centered approach. In the teacher-centered approach to teaching, information flows from the teacher to the learner. Learners become a recipient. In the constructivist classroom, however, the teacher functions as a coach and learners work through problems. Where learners discover concept through activities by themselves, they value mathematics and are able to apply it in real-life. The essence of teaching and learning of mathematics is its applicability in solving daily life problems. The learner-centered approach to teaching inspires students to effectively solve problems by relating various mathematical concepts in real life. The differences between the organization of the constructivist and teacher-centered approach classrooms are presented in Table 1.

Table 1. Differences in the Two Approaches of Teaching Mathematics

Learner-centered classroom		**Teacher-centered classroom**	
i.	Students learn from their own experiences through activities.	i.	Students learn mainly from teachers' explanations
ii.	Students are more responsible for their own learning using the teaching and learning resources under the guide of a teacher	ii.	Students are given more questions to solve using the rules and formulae by a teacher
iii.	Teacher engages all students in lesson; facilitates problem-solving among students.	iii.	Teacher explains the mathematical rules and procedures and gives problems that required the use of such rules
iv.	Teacher establishes a purposeful learning environment, interacts with students, and guides them to construct their own meaning to concept.	iv.	Teachers focus more on procedural understanding
v.	Teachers construct problems carefully that required students' to apply what they learnt	v.	Teachers give problems mostly from textbook that required the use of formula by learners to solve

Effect of a Learner-centered Approach to Teaching on Students' Conceptual Understanding of Mathematics

Some studies have reported the learner-centered approach's effectiveness in teaching and learning mathematics on students' conceptual understanding. Bada (2015) conducted a study on constructivism learning theory for students' understanding of mathematics and contended that if all students are to succeed then teachers have to use learner-centered approach teaching. The study revealed that the learner-centered approach to teaching motivates students to learn and gives teachers the opportunity to assess how the activity is helping the students to develop an understanding of concepts. Furthermore, a study was conducted by Major and Mangope (2012) on constructivist theory in Mathematics in Botswana primary schools. The study focused on the extent to which teachers in Botswana primary schools use the learner-centered approach in teaching and learning of Mathematics. A total of 83 mathematics lessons were videotaped and analyzed. The results revealed that most of the lessons required students to recall rules or formulae. However, small percentages of the lessons required students to explore or investigate mathematical ideas.

Students are at the center of the learning process in the constructivist class through doing, experience, action, and peer collaboration. National Council of Teachers of Mathematics 2000 (cited in Prideaux 2007) contended that when teaching is student, the learners value mathematics and engage actively in learning the discipline. In solving problems under the constructivist strategy, learners are motivated to try various means possible; hence they invent the means of solving the problem themselves. In constructivism, problems are carefully constructed to challenge students to reason about the concepts. Additionally, the interest of the learners increases because they see the need to study mathematics. Mathematics is full of problem-solving. So, employing a problem-solving strategy/technique to teach mathematics helps learners to have a better understanding of mathematical concepts.

Materials and Methods

Design

This study used the nonequivalent (pre-test and post-test) control-group design of quasi-experimental research. In this design, a dependent variable is measured in one group of subjects before and after a treatment and that same dependent variable is measured at pre-test and post-test in another nonequivalent control group which does not receive any treatment. The two groups are selected without random assignment and both groups take a pretest and posttest. But only the experimental group receives the treatment (Creswell & Creswell, 2018). This design makes it possible for researchers to compare scores before and after a treatment in a group that receives the treatment and also in a nonequivalent control group that does not receive the treatment. The nature of the research problem and the objectives to be achieved warranted quasi-experimental design. Importantly, in an environment such as a college of education, randomness was impractical (Leedy & Ormrod, 2020) and despite the argument that quasi-experimental research has low internal validity compared to true experiments, quasi-experimental research has a higher external validity because it helps researchers to do interventions in the real-world instead of artificial laboratory settings (Thomas, 2020).

Population

There are seven public Colleges of Education in the Volta Region of Ghana. Six out of the seven colleges are mixed-sex colleges, and one is female-only. The single-sex college was purposively chosen to ascertain how these students develop their probabilistic reasoning from the two different teaching methods discussed above. The population was all second-year preservice teachers of that college since at the time of the study, the final years were out for their teaching practice and the first year students had not taken the statistics and probability course. This institution was selected for one main reason, they all studied Statistics and Probability and were thus expected to teach it at the Junior High School level upon completion of their training.

Sample and Sampling Procedure

Purposive sampling technique was used to select two intact classes out of six classes identified as Group A and Group B respectively. These groups are similar in terms of their performance in their second-year first-semester mid-semester quiz scores in mathematics with mean scores of 13.9 and 13.7 respectively. “Quasi-experimental designs identify a comparison group that is as similar as possible to the experimental group in terms of pre-intervention characteristics” (White & Sabarwal, 2014, p. 1). Also, it helped to avoid selection bias which is a concern in quasi-experimental studies.

These two intact classes comprised 27 preservice teachers in Group B and 32 in Group A. In purposive sampling, certain elements of the population are deliberately selected on the judgment of the researchers and nothing is left to chance. In this study, respondents were chosen from a population of regular preservice teachers who had studied probability previously and were taking statistics and probability as a course in a college of education. The reason for choosing both groups from the same population is that both Group A and Group B used the same textbooks and course materials. Also, both groups were chosen from the same population to avoid bias because all the colleges use different textbooks and the conditions under which students studied are different from college to college.

Achievement Tests (Pre-test and Post-Test)

The test items were administered in two phases in the classrooms. These are pre-test and the post-test. Items 2i, 2ii, 3i and 3ii of the pretest were adapted from (Anway & Bennett, 2004; Hirsh & O'Donnell, 2001). Two examiners of statistics and probability were consulted to ensure the questions are at the level of the participants. This is to ensure the validity of the test items. The pre-test and the post-test were similar to measure whether there is a change or not. The items were developed to measure the probabilistic reasoning of preservice teachers. The items sought to ascertain preservice teachers’ ability in reasoning about uncertainties and the use of probability daily.

The achievement test assessed the following areas: correctly applying probability in decision making; equiprobability bias; correctly interpreting without replacement situations; correctly listing the sample space and determining the probability; representativeness bias, and interpreting probability statements correctly and applying appropriate strategies. There were 12 items for the pre-test and 10 items for the post-test. We have more items on the pre-test because the post-test was more open-ended than the pre-test. Nevertheless, the total scores in each case was 20. Some of the test items consisted of two parts: the principal question (multiple choice) and a justification of the answer. The pre-test and post-test can be seen in appendices A and B respectively.

Evaluation of Materials by Experts

The researchers solicited two experts' views on the test items, content coverage, content relevance, and course materials. It was the view of the experts that the test items of both the pre-test and the post-test were in line with the mathematics syllabus of colleges of education in Ghana. The activities designed for the Group A were also assessed. In the view of the experts, the teaching activities had sufficient content coverage and would help pre-service teachers develop better probabilistic reasoning. They acknowledged the idea of having activities that reflect real life situations to support pre-service teachers since they were aware students have difficulty with probability related problems. The experts further acknowledged that the objectives were measurable and activities were practical to promote pre-service teachers' involvement in the lessons. They said the teaching and learning materials reflected the learner-centered approach and were in line with the syllabus requirements for Mathematics in colleges of education in Ghana. They believed the activities would promote better probabilistic reasoning since they reflected real life situations.

Data Collection Procedure

Preservice teachers were informed one week ahead of the administration of the pre-test. The format of the test items and the purpose of the study were communicated to the respondents. The tests were administered by the researchers at an Examination Hall to ensure the respondents did independent work.

Treatment

The learner-centered approach was applied to group A while the teacher-centered approach of instruction was applied to group B. In each approach, the lessons were focused on addressing equiprobability bias, representativeness, belief bias, positive recency and negative recency effects since some studies suggest that teaching approaches that target probabilistic misconceptions will help improve students' performance significantly (Khazanov & Prado, 2010; Shay, 2008). The implementation of both teaching approaches were done by the lead researcher.

Group A: The Learner-centered approach

The teaching and learning took place in traditional classroom settings, generally consisting of activities, discussions, and questions and answers concurrently once per week for two hours for a month. In group A, the activities were designed focusing on probabilistic misconceptions using the learner-centered approach to teaching to better their probabilistic reasoning. Examples of the activities are given below.

Card Game: This game was played in groups. Identical cards were numbered 3, 3, 4, 4, 5, 5, 5, 6, 6, 7, 8 and 9 and put in a container, and thoroughly mixed. Three students will reach in and without looking, draw out a card each at time. If the three people pick different numbers, then the group wins GH300.00, else they lose. Is the chance of winning or losing the same, and why? The purpose of this activity is to enhance preservice teachers' decision-making ability by addressing equiprobability bias, representativeness bias, belief, and positive and negative recency effects. The aim of predicting before picking was to keep students focused and to confront any prior misconceptions. The game was played 15 times. If all the three members of the group pick different colors, then a point of one is recorded, else zero. After the activity, a discussion was organized around their prediction and the results to challenge their thinking.

Pair Game: In pairs, preservice teachers were asked to toss a coin six times each, recording the sequence of tails and heads. The purpose of this activity is to address representativeness bias. Before the toss students were asked to predict the number of tails and heads. Their predictions were written in their notebook before the toss. Most preservice teachers predicted equal number of tails and heads. Preservice teachers then compare their predicted sequences to their results after the toss. It was noted in many instances that those who predicted equal number of tails and heads did not obtain results that were consistent with their prediction. Discussions were organized around their predicted sequences and the results obtained after the toss for better probabilistic reasoning.

Coin Game: Students perform this experiment ten times by tossing a number of fair coins once and record the outcomes in each case. In the first toss, a coin was tossed once and the outcome recorded, in the second toss, two coins were tossed once and outcomes recorded, in third toss, three coins were tossed once and outcomes recorded, and so on. H stands for heads and T stands for tails. The following are a sample of sequence of outcomes by one student.

1st toss: H
2nd toss: HT
3rd toss: HTH
4th toss: HTTT
5th toss: THHTT
6th toss: HTHTTT
7th toss: TTHTHHT
8th toss: THTHTHHH
9th toss: HTTTHTHTT
10th toss: THTHTHTHTT

$$\frac{Number\ of\ heads}{Number\ of\ tosses} = \frac{1}{1} = 1;\ \frac{1}{2} = 0.5;\ \frac{2}{3} = 0.667;\ \frac{1}{4} = 0.25;\ \frac{2}{5} = 0.4;\ \frac{2}{6} = 0.333;\ \frac{3}{7} = 0.429;\ \frac{5}{8} = 0.625;\ \frac{3}{9} = 0.333;\ \frac{4}{10} = 0.4$$

The average of the probabilities of obtaining a head for each experiment is

$$= \frac{1+0.5+0.667+0.25+0.4+0.333+0.429+0.625+0.333+0.4}{10} = \frac{4.937}{10} \cong 0.5$$

It must be noted that in experimental or frequentist probability, we are concerned about conducting the experiment many times and considering its approximation or convergence. Students were led to conclude that in general, we can say when a fair coin is tossed, the probability of getting a head at any time is $\frac{1}{2}$ and vice versa.

Group B: The Teacher-centered approach

Group B was taught using the teacher-centered approach. In this case the teacher, rather than the learner, was at the center of the teaching and learning process. The tutor presented the preservice teachers with facts or procedures that had been established to solve problems. Preservice teachers were given problems that require the use of these facts or procedures to solve as either assignment, exercise, test, or quiz. It is believed that preservice teachers learned more when they solved similar problems many times. When examination questions that demanded facts were set, students could solve the problems without really understanding the concepts. Formulae were presented to students to learn to apply to similar problems. Concepts were clearly explained bearing in mind the probabilistic misconceptions. Samples of lessons experienced by Group B are as follow.

Lesson one: Through questions and answers, teacher revises preservice teachers' previous knowledge on playing of games and probabilistic situations. Based on students' responses, the teacher introduces the lesson.
Teacher explains to preservice teachers the concept of probability and three ways we can make probability statements as classical, frequentist and subjective. Classical probability is the number of favorable outcomes (E) divided by the total number of possible outcomes (S). It works on the assumptions that all outcomes are equally likely. Equally likely means that each outcome has the same probability of occurring.
Mathematically, P(E) = $\frac{N(E)}{N(S)}$. Students were then led to solve problems using the formula.

Experimental probability is a method of calculating probability where the probability is the ratio of the number of the trials where the desired event occurs to the number of total trials. Subjective probability is a statement that an individual makes based on personal judgment to describe the likelihood of an event. Or a personal belief based on one's own experience or expertise. The person making the statement should be an expert in the area or someone who have a sound knowledge on the issue under consideration before it can be considered important. For the statement to be reliable, it should be based on a sound premise(s). Nevertheless, both experts and non-experts make use of subjective probability daily. For example, I have a 90% chance of passing my statistics and probability exam.

Lesson two: Teacher explains basic terms of probability such as sample space, sample point, an event, equally likely to preservice teachers. Teacher guides preservice teachers through demonstration to solve probability problems using classical definition of probability by listing the sample space and finding the probability of an event. Teacher leads preservice teachers to solve probability problems from frequency tables. Similarly, teacher explains to preservice teachers independent and dependent events. Teacher asks the preservice teachers to solve problems on independent and dependent events.

Data Analysis

The data collected were coded and edited before being transferred to computer for analysis using SPSS version 24. A Levene's test was used in the study to ensure that the assumptions between each group was not violated. Homogeneity of variances was considered to ensure that each group's pre-test scores were mostly equal. Paired samples T-test was used to compare two means within each group to determine whether or not the differences in means were statistically significant. Independent sample T-test was used to compare the probabilistic reasoning mean scores in Group A and the Group B before the experiment at a time to determine whether the two groups were at the same level or not. The independent t-test was used because there were two different groups of participants. It works on the assumption that scores are unique because they come from different participants. Also, the equality of variance for running ANCOVA at 5% significance level was checked using Levene's test.

Table 2. Levene's Test of Equality of Variances

F	df1	df2	Sig.
1.234	1	57	0.456

The results from Table 2 indicate that the variances are roughly equal since the significance value of 0.456 is greater than the cut-off value of 0.05. Therefore, we can conclude that homogeneity of variance assumption was met (Field, 2005). This allowed us to examine the relationship between the covariate (pre-test scores) and dependent variable (post-test scores). Analysis of covariance (ANCOVA) was used to examine the effect of learner-centered approach to teaching and teacher-centered approach on preservice teachers' probabilistic reasoning since different participants were in each group. The ANCOVA is ideal in removing the bias of these variables (Field, 2005). In order to determine whether or not the occurrence of scores was not by chance, a partial eta squared was used to gauge the magnitude of the difference (effect size) between means of the scores. Pallant (2001) outlined the following criterion for interpreting partial eta squared values as 0.01 = small effect, 0.06 = moderate effect and 0.14 = large effect.

Results

Before introducing the learner-centered and teacher-centered approaches of teaching, preservice teachers' probabilistic reasoning was compared to ascertain whether or not their results were similar in terms of the mean scores. The result is presented in Table 3.

Table 3. Results of the Independent Samples t-test on the Pre-test of the Group B and Group A

Group	*N*	Mean	Std. Dev.	*Mean Difference*	*Df*	t-value	*p*-value
Group B	27	5.85	1.562	.35	56	.756	.456
Group A	32	5.50	1.967				

The results in Table 3 indicate that there was no statistically significant difference between the probabilistic reasoning mean scores of Group A ($M = 5.50, SD = 1.967$) and Group B ($M = 5.85, SD = 1.562$) in the pretest. This is an indication that the two groups are similar and their performance could be compared after the two teaching approaches.

Comparison of Probabilistic Reasoning of the Preservice Teachers before and after Learner-centered Approach in Group A

The results of analysis of the effect of the learner-centered approach on preservice teachers' probabilistic reasoning is presented in Table 4.

Table 4. Pre-test and Post-test Scores for Group A

	Minimum	**Maximum**	**Mean**	**Std. Deviation**
Pre-test	1	10	5.50	1.967
Post-test	8	20	15.72	3.630

Table 4 reveals that in the post-test, preservice teachers' probabilistic reasoning mean scores increased from 5.50 to 15.72 in the post-test in Group A.

A paired sample *t*-test was conducted to compare the post-test and pre-test scores for the preservice teachers taught with the learner-centered approach to teaching (Group A) in order to determine if the mean difference in scores was statistically significant. This was done to evaluate the effect of the learner-centered approach on preservice teachers' probabilistic reasoning. Table 5 presents the results.

Table 5. Paired Sample t-test Results on the Post-test and Pre-test of Group A

	Mean Difference	**Std. Deviation**	**Std. Error Mean**	***t***	***Df***	**Sig.**
Post-test – Pre-test	10.219	2.871	.507	20.137	31	.000

The results from Table 5 shows a statistically significant difference in the preservice teachers' probabilistic reasoning from the pre-test (M = 5.50, SD = 1.967) to the post-test (M = 15.72, SD = 3.630), with t (31) = 20.137, p = .000. The result reveals that preservice teachers' probabilistic reasoning has improved significantly after the learner-centered approach was used, since $p = .000 < .05$. It can be inferred that the learner-centered approach to teaching has a positive significant effect on preservice teachers' probabilistic reasoning.

Analysis of Probabilistic Reasoning of the Preservice Teachers before and after Teacher-centered Approach to Teaching

The results of the analysis of the effect of the teacher-centered approach to teaching on preservice teachers' probabilistic reasoning is presented in Table 6.

Table 6. Pre-test and Post-test Scores for the Group B

	Minimum	**Maximum**	**Mean**	**Std. Deviation**
Pre-test	2	8	5.85	1.562
Post-test	5	14	9.52	2.208

The data on Table 6 show that the mean score of preservice teachers in the pre-test was 5.85, while that of the post-test was 9.52 marks. This indicates that in the post-test, an average of each of the preservice teachers' probabilistic reasoning increased slightly in Group B. To determine if the mean difference is statistically significant, a paired sample t-test was conducted. Table 7 presents the results.

Table 7. Results of the Paired Samples t-test on the Post-test and Pre-test of the Group B

	Mean Difference	**Std. Deviation**	**Std. Error Mean**	***t***	***Df***	**Sig.**
Post-test – Pre-test	3.667	1.441	0.277	13.220	26	.000

The result presented on Table 7 shows that the mean score difference between the post-test and pre-test of Group B was 3.667 marks with a corresponding standard deviation of 1.441. The paired sample t-test was conducted to find out if the mean score difference (M = 3.667, SD = 1.441) was statistically significant. This was done to assess the effect of the teacher-centered approach targeted at addressing misconceptions on preservice teachers' probabilistic reasoning. The results from Table 7 reveal that there was a statistically significant difference in the preservice teachers' probabilistic reasoning scores from the pre-test (M = 5.85, SD = 1.562) to the post-test (M = 9.52, SD = 2.208) with t (26) = 13.220, p = .000 < .05. This is an indication that the teacher-centered approach to teaching probability targeted at addressing probabilistic misconceptions aids students to improve upon their understanding and problem-solving abilities in probability.

Next, we compared the effectiveness of the learner-centered approach of teaching and the teacher-centered approach of teaching in terms of probabilistic reasoning of preservice teachers and how they addressed probabilistic misconceptions.

Comparison of Effectiveness of Learner-centered Approach and Teacher-centered Approach in Developing Probabilistic Reasoning

The preservice teachers' scores on the pre-test of the achievement test were used as covariate in this analysis. The scores from the post-test were then compared using ANCOVA with $\alpha = .05$, to adjust for pre-test differences that existed between Group A and Group B. In the pre-test the mean scores of Group B and Group A were 5.85 and 5.50 respectively. ANCOVA was used to adjust the post-test scores statistically to compensate for the 0.35 mean point difference between the two groups in the pre-test. This adjustment led to a more accurate post-test comparison. Table 8 presents the results.

Table 8. Summary of ANCOVA of the Performance of Preservice Teachers taught with Learner-centered Approach and Teacher-centered Approach (*N* = 59)

Source	**Sum of Squares**	***Df***	**Mean Square**	***F***	**Sig.**	**Partial Eta Squared**
Corrected Model	791.144[a]	2	395.562	72.150	.000	.720
Intercept	205.135	1	205.135	37.416	.000	.401
Pre-test	228.184	1	228.184	41.620	.000	.426
Group	630.253	1	630.253	114.955	.000	.672
Error	307.026	56	5.483			
Total	10888.000	59				
Corrected Total	1098.169	58				

a. R Squared = .720 (Adjusted R Squared = .710)

The data on Table 8 indicate that there is a statistically significant main effect on the preservice teachers who were taught using the learner-centered approach to learn probability, [F (1, 56) = 114.955; p = .000 < .05]. Consequently, the preservice teachers taught with the learner-centered approach performed better than their counterparts who experienced the teacher-centered approach to learn probability. The partial eta squared value was found to be 0.672. This value indicates a large effect size. That is, 67.2% of the variance in the dependent variable (probabilistic reasoning achievement score) is expounded by the independent variable (teacher-centered approach). This implies that the magnitude of the difference between the mean score of the preservice teachers taught with the learner-centered approach and the teacher-centered approach is large. With the high scores of preservice teachers in the Group A, it implies that the use of learner-centered approach in the teaching and learning of probability improved the performance of the preservice teachers.

The outcome of the study portrays that there was a statistically significant difference in the probabilistic reasoning achievement mean scores of Group A than that of Group B [F (1, 56) = 114.955; p = .000 < .05]. This finding implies that Group A performed better than Group B in the probabilistic reasoning achievement test. This is an indication that when the learner-centered approach to teaching probability is targeted at addressing probabilistic misconceptions, preservice teachers develop better probabilistic reasoning than teacher-centered approach.

Discussion

This study set out to compare the learner-centered and teacher-centered approaches of teaching in order to determine the best way of developing preservice teachers’ probabilistic reasoning at the College of Education level in Ghana. The results, as shown on Tables 3, 4, 5, 6, 7 and 8 indicate that the learner-centered approach to teaching probability is more effective than the teacher-centered approach in motivating and inspiring preservice teachers to learn and apply probability. The learner-centered approach provides students' opportunities to think, reason, reflect and evaluate their ideals and that of colleagues.

The findings of the present study are in tandem with several research findings. Lessani, Md. Yunus and Bakar (2017) investigated mathematics teaching approaches and their consequences on learners' ability to solve challenging problems. Their findings affirm the learner-centered approach to be more effective in students' creativity and innovation in solving real-life problems than the teacher-centered approach. So, training programs for excellent teaching and learning should be flexible and innovative to allow preservice teachers to explore gaps in learning and teaching (AIAhmad, 2021). Similarly, Prideaux (2007) found that students who experienced the learner-centered approach of teaching performed significantly higher than those who were taught without the learner-centered approach of teaching. Roseth, Garfield and Ben-Zvi (2008) also examined the effect of the cooperative framework in classroom teaching and collaboration and found that student-centered approaches to teaching help students to have better understanding of statistics than teacher-centered instruction.

Furthermore, Gurbuz and Birgin (2011) studied the effect of computer-assisted teaching on addressing probabilistic misconceptions of students. The authors used pre-test to collect data on both the computer-assisted and traditional groups before the experimental teachings. After the intervention, the post-test was administered to ascertain the effect of computer-assisted teaching and teacher-centered approach on addressing students' probabilistic misconceptions. Their study found computer-assisted teaching to have more significant effect on students' correct probabilistic reasoning than teacher-centered approach of teaching.

As noted by Batanero and Diaz (2012), probability forms an integral part of the Mathematics curriculum in many countries. But the concept is one that some learners find difficult to understand partly due to the method of teaching used by teachers. This fact implies that teachers of mathematics should find more creative ways of teaching the concept so as to make it easier for learners to understand and apply the concept. As discovered in this study, the learner-centered approach is effective in developing probabilistic reasoning. But the learner-centered approach works best when certain conditions are present. At the outset, there is the need for relevant teaching and learning materials. Then teachers need to be trained and retrained in the use of the learner-centered approach to teaching. At the college of education level, preservice teachers need to be taught using the learner-centered approach not only to help them understand probability but also to help them use the learner-centered approach in their own teaching.

The activities used for Group A, which is learner-centered approach group in the form of games, promote content knowledge and cognitive skills competencies such as collaboration and problem solving (Belova & Zowada, 2020). The activities focus on promoting learning experiences rather than stressing on the entertainment factor. While some things students learn in school seem to them predetermined and bound by laws in other branches of mathematics, probability provides students with opportunity to learn that solutions to several problems depend on assumptions and have degrees of doubt. So teaching methods that allow students to predict and test their predictions should be encouraged for students to develop their intellectual repertoire. The processes used in reasoning about sample space will serve students well in life. Several calls on the need to pay attention to the study of probability show that teachers and mathematics teachers must provide their students with appropriate teaching strategies to enhance their conceptual understanding. In the preparation of preservice teachers to become mathematically competent citizens, teacher preparation programs such as teaching methods must provide opportunities for preservice teachers to confront misconceptions and reason probabilistically (Courtney & Caniglia, 2021).

The findings of the present study, however, run counter to those of Weltman and Whiteside (2010). They conducted a study on the effectiveness of the traditional and active learning method in business statistics. The study found that active learning had not been effective for all category of students. It is not precisely clear what may have accounted for that result. This is because active learning methods have generally been known to be effective in promoting learning. Further research is thus required in this regard.

Conclusions

This study compared the effects of the learner-centered and teacher-centered approaches of teaching in addressing probabilistic misconceptions by measuring preservice teachers' probabilistic reasoning scores. The results show that the learner-centered approach aids preservice teachers to develop better probabilistic reasoning than the teacher-centered approach to teaching. Thus, the study concludes that the learner-centered approach to teaching is a solution to preservice teachers' inability to answer questions on probability correctly. Even though each teaching approach targeted at addressing misconceptions was found to have significant effect on pre-service teachers' probabilistic reasoning, the learner-centered approach was found to be more effective than the teacher-centered approach in addressing misconceptions which limit students' ability to solve probability problems correctly.

Implications and Recommendations

This study makes some key contributions to the teacher education/development literature in general, and particularly to the development of probabilistic reasoning of preservice teachers. The study provides empirical evidence that the learner-centered approach targeted at addressing probabilistic misconceptions is more effective in developing probabilistic reasoning in preservice teachers. What this means is that when teacher educators use the learner-centered approach to teach the concept of probability and focus on addressing misconceptions, preservice teachers will develop better probabilistic reasoning. This will, in turn, translate into preservice teachers answering questions on probability correctly in their examinations and ultimately making them teach the concept more effectively when they complete their training. It is thus recommended that teachers should consider employing the learner-centered approach in teaching the concept of probability.

Limitations

As with every empirical study employing the quasi-experimental approach, the current study has some limitations which need to be acknowledged and used to guide users of the findings. Firstly, our sample was drawn from a population of second year preservice teachers of a female only college of education in a peri-urban area in Ghana. Further experiments are therefore needed in other settings in order to see if the same or similar results will occur. Secondly, owing to the narrow characteristics of the participants in the present study, the researchers cannot generalize the results to other individuals who do not have the same characteristics. So, additional experiments with groups with different characteristics are necessary. Lastly, the experiments need to be repeated at another time to determine if the results are the same (consistent) over time.

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Appendix A. Pre-test: Probabilistic Reasoning Assessment

This assessment is for research purpose to improve teaching and learning of probability.

You are required to answer all questions honestly. Do and leave all workings or rough work on the paper.

Pre-test: Probabilistic Reasoning Assessment: Duration: 1hr

Index number……………………………………………Level…………..Age………

Read each statement carefully and **circle** the letter of the correct (best) option. Provide appropriate response in the **space provided** under some items as it apply.

1. If you rolled an ordinary die and obtained 51246, which of these is **most likely** to occur on your next roll?
   a. 1
   b. 2
   c. 3
   d. 4
   e. 5
   f. 6
   g. The options a, b, c, d, e, and f are all equally.

2i. A fair coin is tossed, and it lands tail. The coin is to be tossed a second time. What is the probability that the second toss will also be a tail?

a. $\frac{1}{4}$
b. $\frac{1}{2}$
c. $\frac{1}{3}$
d. Slightly less than $\frac{1}{2}$
e. Slightly more than $\frac{1}{2}$
f. 0

2ii. Which of the following best describes the reason for the correct answer to the preceding question (2i).

a. The second toss is less likely to be tail because the first toss was tail.
b. There are four possible outcomes when you toss a coin twice. Getting two tails is only one of them.
c. The chance of getting heads or tails on any one toss is always $\frac{1}{2}$
d. There are three possible outcomes when you toss a coin twice. Getting two tails is only one of the them.
e. No chance of being a tail in the second toss

3i. A box contains 6 balls: 2 are red, 2 are black, and 2 blue. Three balls are picked at random, one at a time. Each time a ball is picked, the colour is recorded, and the ball is put back in the box, mixed thoroughly. If the first 2 balls are black, what color is the third ball **most likely** to be?

a. Red

b. Black

c. Blue

d. Red and blue are equally likely than black.

e. Red, black and blue are all equally likely.

3ii. Which of the following best describes the reason for the correct answer in the preceding question (3i)?

a. The third ball should not be black because too many black ones have already been picked.

b. The picks are independent, so every color has an equally likely chance of being picked.

c. Black seems to be lucky.

d. This color is just as likely as any other color.

4. A spinner with ten equally divided sectors in which four parts are white and the rests are red is about to be spin, which color is the spinner **more likely** to land?

    a. White

    b. Red

    c. Both white and red are all equally.

5. If you toss a fair coin and get heads two times in a row, which of these is **most likely** on next toss?

    a. Tail

    b. Head

    c. Both head and tail are all equally.

6. A Gh¢2, Gh¢5, Gh¢10, Gh¢20, Gh¢50, and Gh¢100 notes were put in a box and thoroughly mixed. What is the probability of picking Gh¢50 note without looking?

    a. $\frac{1}{187}$

    b. $\frac{50}{187}$

    c. $\frac{1}{6}$

    d. 0

7. In a game you have been offered two options. Which of the following give you higher chance of winning GH¢200.

    a. Tossing one die once and obtaining a 6

    b. Tossing two dice once and obtaining two 6's

    c. The options a and b are all equally

8. A fair die is to be toss six times, which of these is **most likely** to occur?

    a. 251634

    b. 123456

    c. 666341

    d. 132244

    e. The options a, b, c, and d are all equally likely

9. A fair coin is to be toss four times in a row, which of these is **least likely** to occur?

    a. THTH

    b. TTTH

    c. HHH

d. The options a, b and c are all equally likely?

10. Two fair dice are to be tossed once. Which of the following is **more likely** to occur:
    a. Obtaining a sum of 11.
    b. Obtaining a sum of 10.
    c. A sum of 11, and a sum of 10 are all equally likely.

10i. Give reason for the correct answer in the preceding question (10).

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11. In national lottery, one has to choose five numbers from 1 to 90 inclusive. You win when at least two of your numbers are part of the numbers drawn on Saturday at random. Eric has chosen 4, 6, 7, 9, and 11. Alex has chosen 15, 23, 45, 66, and 80. Daniel has chosen 71, 77, 78, 83, and 85. Do Eric, Alex, and Daniel have the same chance of winning or not, and why?

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12. A box contains 8 yellow balls and 4 green balls. Two balls are chosen at random, one after the other, without replacement, determine the probability that the two balls are of different colours.

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Appendix B. Post-test: Probabilistic Reasoning Assessment

This assessment is for research purpose to improve teaching and learning of probability.

You are required to answer all questions honestly. Do and leave all workings or rough work on the paper.

Post-test: Probabilistic Reasoning Assessment. Duration: One Hour

Index number……………………………………….Class..…………..Age…………

Read each statement carefully and **circle** the letter of the correct (best) option for each item as much as possible and **provide** appropriate response in the space provided.

1. Suppose a particular outcome from a random event has a probability of 0.4. Which of the following statements represents correct interpretations of this probability?
    a. The outcome will not happen
    b. The outcome will certainly happen about four times out of every 10 trials.
    c. The outcome is expected to happen about four times out of every 10 trials.
    d. The outcome could happen, or it couldn't, of either result are the same.
2. If you toss a fair coin and get heads 3 times in a row, what is the chance of getting a head on the next toss?
    a. 1
    b. Greater than $\frac{1}{2}$
    c. $\frac{1}{2}$
    d. Less than $\frac{1}{2}$
    e. 0
3. A die is numbered 1, 2, 3, 4, 4, 5. When the die is thrown once. Which of these is most likely and why?
    a. 1
    b. 2
    c. 3
    d. 4
    e. 5
    f. They are all equally likely

3i. Give reason for your chosen answer in (3).

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4. German MP tossed two dice once. Which of these is mostly likely and why?
    a. Obtaining a sum of 4.
    b. Obtaining a sum of 7
    c. Obtaining a sum of 11
    d. All of them have equal chance of occurring that is a, b, c.

4i. Give reason for your chosen answer in (4).

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5i. A fair coin is to be toss four times, which of these is **most likely** to occur?

a. HTHT
b. THTT
c. TTTT
d. HHTH
e. The options a, b, c, and d are all equally likely

5ii Give reason for your chosen answer in (5i).
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6i. A fair coin is to be toss four times, which of these is **least likely** to occur?

a. HTHT
b. THTT
c. TTTT
d. HHTH
e. The options a, b, c, and d are all equally likely

6ii. Give reason for your chosen answer in (6i)

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7. A college has 300 level 200 students and 236 level 100 students in it. Each student's name is written with the level on a piece of paper and all the names are put into a box and mixed thoroughly. A tutor picks one name out of the box without looking. Which level is the tutor more likely to pick from and why?

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8. Suppose that you toss a coin three times and all eight outcomes are equally likely. If it is known that the first toss comes up heads, what is the probability of an odd number of heads appearing?

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9. A card is selected at random from a pack of 52 cards. Find the probability of selecting a card which is a red or a queen.

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10. A box contains 6 white balls and 4 green balls. Two balls are chosen at random, one after the other, without replacement, determine the probability that the second ball is white.

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