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Mathematical Modelling with Engineering Students PBL Methodology and Proposal of a Quantitative Evaluation Rubric

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Abstract

The implementation of a PBL pedagogical experiment intended to understand how 2nd year students of engineering programs of Aeronautics and Materials Sciences applied mathematical contents to problem solving, specifically as regards geometrical modelling of an object and calculating its volume and centre of mass resorting to triple integrals. A qualitative methodology was used, this being a study whose importance consisted in a pedagogical experiment, which drove students to problematization, research and interdisciplinarity. At the end, we can conclude that this experiment has contributed to fostering the motivation and efficiency of significant learning of calculus contents. Students considered they were stimulated to use contents taught in class, deeming it an excellent initiative, which led them to escape the usual format of teaching and proved to be quite effective in grasping and applying knowledge, an initiative that should be maintained. They also stated they had acquired tools that could help them to overcome future difficulties and that their awareness level regarding application of theory to practice increased. The construction of a quantitative evaluation rubric, which is applied, is also presented, and resulted in an 81% grade for the undertaken activity.

Introduction

Different ways of teaching and learning constitute a subject of debate in education community, spanning all schooling levels. As regards mathematics education, thinking of it as a set of knowledge and skills, albeit not wrong, it is not suitable for school mathematics today, since, as Li (2019) clarifies, the quantity of concepts and procedures after years of development surpasses what might be covered in any curriculum. Thus, nowadays instead of accumulating concepts, it seems to make more sense to opt for the study of some of them, namely those applicable to real life situations, in face of a society that demands proactiveness.

According to Dyke (2015), problem based learning [PBL] is a teaching learning methodology that aims to develop in students problem-solving skills. Moreover, Yeung et al (2003) add that it is a way of learning that fosters a deeper understanding of the taught content, promotes teamwork, research and the collection of credible information. PBL is seen as an efficient method, especially in practical, realistic demonstration of concepts that often lie in an abstract domain. In fact, according to Fatokun and Fatokun (2013) the feature that sets PBL apart from the rest as teaching technique, education strategy or even philosophy, is the change in the whole learning environment required by the technique itself. According to these scholars, defining PBL as an educational philosophy means holistically consider all the elements of teaching: organizational context, curricular content and its design, the approach to teaching and learning and the evaluation method.

Theoretical Framework

According to Hidayah et al (2020), teaching by the PBL method involves orienting students to the problem, organizing them to learn, supporting the individual and group research, developing and presenting work, analysing and evaluating the process used and the solution to the problem. The method proposed by these authors is designed to stimulate critical thinking directed at the submitted problem and it is based on five stages: analysis, structuring, development, implementation and evaluation. In their view, this model can help create interest in the learning of (mathematical) calculus since it is based on the proposal to solve an actual problem, and it develops critical thinking skills. At the same time, it helps to reduce the common difficulties in learning this type of contents as well as overcome the monotony of traditional classes. From its origin, the PBL methodology has been constantly research, in several studies, like those carried out by Ruikar and Demian (2013) or Gibbes and Carson (2014). The results of meta-analysis done by these authors point to significant gains for the students, such as: definition of intellectual challenges, flexibility and overcoming obstacles, collaboration and team spirit, and work under pressure. The literature reinforces that the use of PBL enables the emergence of tools and skills which students would not be able to acquire with the use of the standard method. Furthermore, it argues that the use of PBL as a tool for consolidating content allows students to develop disciplined autonomous work and overcome adversities (Daher & Anabousy, 2020; Govaris, 2011; Hartono & Ozturk, 2022; Jackowicz & Ozturk, 2021; Vaux, Moore, & Nordhues, 2022;). Thus, the advantages of using a PBL approach to support the experiment constructed and applied in this study have been clearly identified.

Although this approach to teaching and learning is more disseminated among students not yet attending higher education. Some documented didactical instances can be found regarding the implementation among students attending bachelor programmes, namely in the teaching of calculus, in curricular units in the scientific field of mathematics. According to Cuzzuol et al (2018), in studies carried out, the learning process in the subject of calculus proved to be more effective with the use of the PBL methodology, and it was remarked that this method made learning of a difficult subject a more interesting and pleasant task for the students. Fatokun and Fatokun (2013) state that to create a problem to be proposed to the students as a challenge, it must be well defined so that it will enable them to grasp the curricular contents they are meant to acquire, to use or to develop. According to these authors, PBL, when applied to mathematical calculus and most scientific matters, must be thought out taking in consideration the importance of identifying a specific area for the basic concepts, personalizing knowledge, capturing the need for and the sense of rigour in both written and oral expression, fostering the need for abstraction and using it appropriately, proving, generalizing and criticizing results, modelling different situations by using suitable mathematical tools, interpreting and assessing the obtained results. They also argue that the orientation given to the students by the teacher should not hinder their initiative to research and expand the problem, taking into account that there may be several approaches to solving them.

For Hidayah et al (2020) the PBL methodology focuses on the students and its advantage is to motivate them to expand their knowledge and abilities. With this method, students learn in a small group or individually, becoming partners in the teaching learning process, capable of facing new situations and developing skills that will serve them for the rest of their lives. Blass and Brasil (2020) go deeper. They claim that this method is a viable option for teaching calculus contents in engineering, besides being a challenge for the professional development of the teachers applying it, as it involves careful preparation of the method and the problems submitted so as to ensure, as much as possible, the desired contextualization of the contents covered.

The study presented in this text combines the PBL methodology with mathematical modelling. Despite the possibility of defining modelling by the word *method*, derived from Applied Mathematics, its main feature is the translation of a content into mathematical language. The development of a mathematical modelling project is grounded, according to Al-Balushi and Al-Aamri (2014), on fostering students' critical participation in issues that refer to present reality. In addition, because in the real-world situations do not occur isolated but rather integrated in contexts of an ecosystemic nature, we believe mathematical content should be associated with contents from other areas of knowledge, thus contributing to foster interdisciplinarity. Following Widjaya et al (2019), in higher education, the curricular units that integrate a particular programme often remain divided into subjects with no trace of link between them. Frequently constituted by rigid syllabuses and traditional teaching and evaluation methodologies, they are taught isolated and not as interdisciplinary practices. In this way, classes are merely a transposition of contents, exercises and techniques, or even merely lectures on theorems and demonstrations deprived of goals, which in some way refer to actual practical cases.

We believe that applying mathematical modelling through pedagogic activities allows the creation of a learning environment in which students are invited to research problematic situations derived from actual reality, and acquire and apply contents as well as establish links between the different curricular units. The digital medium, by itself, arouses young people's interest. According to Kerremans et al (2003) the educational software can be important to obtain pedagogical tools for the teaching-learning process, so that the use of these resources signs a way to boost teaching and motivation to learn different areas, such as mathematics. Many digital resources have the possibility to provide new discoveries, expediting numerical algorithms and thus enabling the focus to be placed on the process as well as on the observation and analysis of results. For this reason, the use of digital tools will be stimulated in the activity created in this pedagogical experiment.

PBL has been implemented with great success in different areas of education, science and medicine. Therefore, there is great need to assess the level of quality and achievement of the challenge derived from using it. Nevertheless, often, conducting that assessment becomes an obstacle to its implementation, due to the lack of information and feasible methods for a fair and balanced assessment (Karaçalli and Korur, 2014). Among scarse evaluation methods, the existence of quantitative models are not observed, only qualitative. One of these is mentioned below, called 3C3R, was proposed by Hung (2006), and is illustrated in Figure 1.

As stated by this researcher, the three main criteria necessary to assess a PBL task, according to the 3C3R model are: content, context and connection. Contents should take into account the curricular patterns, followed by the identification of concepts and topic-related areas, be succinct and avoid ambiguity. Targets or goals should be defined so that students can expand their understanding. Contextual information covers the connection between problems and students, making them acquire knowledge and skills in real life situations. The purpose of the connection element is to articulate concepts and information with content in context.

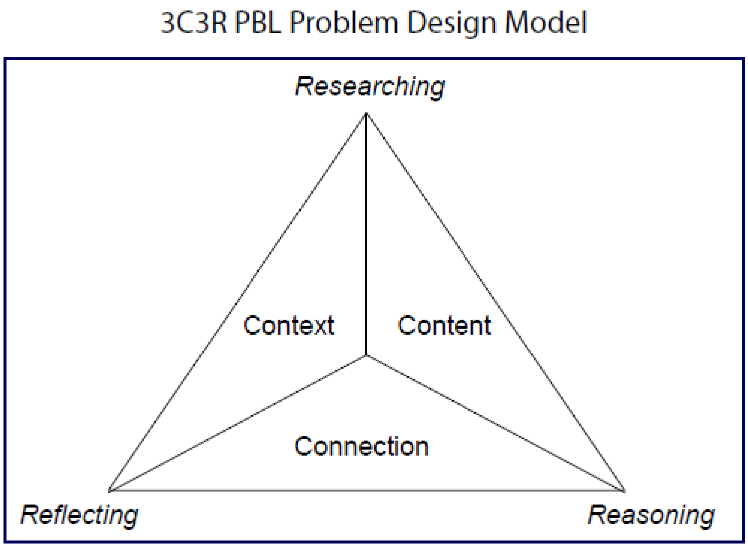

Figure 1. DP – 3C3R Evaluation Model, proposed by Hung (2006)

Applying this evaluation method does not provide a quantitative result, merely a qualitative perception of students' performance, classifying the task as successful or failure. For this reason, we created a rubric, described later, of a quantitative evaluation for a PBL task, which we applied to the implemented pedagogical activity, upon which the study presented in this paper focuses. Next, we will explain the study carried out.

Pedagogical Challenge: PBL and Modeling

Rezende and Silva-Salse (2021) argue that PBL, as didactic tool in the teaching of mathematics, reinforces the development of involved learners' critical thinking. On other hand, with a view to assessing the contribute of PBL type tasks to the learning of mathematics, Magalhães et al (2019) implemented an experiment with students of the 1st year of engineering programmes, and results suggest that students became more motivated and acknowledged the usefulness of this kind of tasks in their academic and future professional life. Besides, the study concluded that there is better management of PBL tasks when they are part of several curricular units. In view of this evidence, we decided to formulate the following research question: How will PBL methodology (with interdisciplinary characteristics) impacts the motivation and efficacy of the meaningful learning of calculus contents when applied to engineering students?

In curricular unit of *Calculus 3*, students were proposed three pedagogical challenges with a view to exploring and consolidating some curricular contents in different moments of the academic semester. One of the authors of the present paper is the lecturer of the curricular unit in question. These contents had been previously taught using a teaching method based on the presentation of concepts and practice of traditional exercises.

In this text we describe the pedagogical experiment undertaken with the first challenge. The general goal aimed to understand how 2^{nd} year students of programmes in Engineering of Aeronautic Sciences and Material apply mathematical contents to problem solving. The specific goal was defined by understanding, in particular, how these students apply geometric modelling to an object and calculate its volume and centre of mass, using triple integrals.

The design and implementation of the pedagogical challenge [PC] went through four stages: (1) design by the lecturer, (2) presentation of the proposal to the students and its resolution, (3) presentation by the students to the group-class and discussion and (4) students final reflections. The lecturer prepared a PC, which she then submitted to the students, starting from an initial problem. As shown in figure 2, students had at their disposal scales, a ruler or a set square and an object which had been allocated to them.

Figure 2. PC – Applied Integral Calculus

In the formulation of the PC could be read: "For each of the five objects presented, consider that the mass density is constant in all points of the object. Use scales to weigh your object, and answer the following questions using integral calculations.
Q1. What is the mass density function?
Q2. What are the coordinates of the centre of mass? Illustrate clearly where it is located.
Q3. What is the total surface area of the object?
Q4. Please reflect on an extension to the proposed challenge.
NB: Start by defining a geometric model as close as possible to the real object and place it on a three-dimensional reference. You can use an *app* that calculates integrals, but you cannot use area or volume formulas.

Since there is an intention behind the activity prepared by the lecturer, Table 1 summarizes the curricular contents to be applied, the scientific knowledge to discover, and the skills to develop. The question was presented to the students after theoretical curricular contents needed for its basic resolution had been taught. Students were free to work individually or in pedagogical pairs. The objects were allocated to the students by the lecturer, taking into account the students' performance so far and the difficulty inherent to each object. It should be pointed out that lecturer had been following these students for three semesters. This allocation allowed all students to initiate the challenge and to feel motivated to commit to the task.

Table 1. Profile of PC1

curricular contents to be applied	scientific knowledge to discover	skills to develop
Calculation of an area using a triple integral	Concept of three-dimensional geometric model	Deciding where to place the three-dimensional referential in the object under consideration
Calculation of a volume using a triple or double integral	Concept of mass density	Using digital tools to expedite algebraic or numeric calculations
Definition and meaning of the centre of mass of a body	function	Finds topics in the field of physics that relate to these, from the field of mathematics

On the day the task was presented, the two-hours class was devoted to tutorial guidance, so the students would clearly understand the question and how to begin exploring the problem given to them. Students were told that, besides answering the questions submitted, they would have to think of an extension to their problem. After this class, work proceeded autonomously, with occasional queries emerging. Students had two weeks to carry out the task and prepare their presentation. They used a Power Point file to support the presentations, in which they included: algebraic resolutions, images and links to digital resources. Students were given about 15 minutes to present their work, with no more than seven slides per presentation (a condition set by the lecturer). Presentations consisted in explaining how students interpreted the problem given to them, how they solved it, how they understood it, and the tools, physical or digital, they resorted to.

After the students had made their presentations to the class, the lecturer set some questions for them to help them understand the actual meaning of the resolution to the problem they had been received. These were those questions:

> "Q1. If an object were filled half with a substance and half with another substance, what changes would there be in the resolution of the challenge?
>
> Q2. How and why did you decide to place the cartesian frame of reference in that position relative to the object? What would happen if you placed it differently?
>
> Q3. If this was your case, why did you decide to change coordinates to calculate the integral and did not use cartesian coordinates?
>
> Q4. What are your thoughts when you compare the exercises done in class before this challenge with the problem given to you by this challenge?"

In this way, the format of this PBL task included a set of four questions when the PC was presented, and a set of four questions for reflection, presented after the projects had been discussed with the group in class.

Method

Regarding the research design, we used a qualitative methodology, common in education, especially in a case study (Amado, 2014). The aim was not to reach generalization but rather the specificities of the case at hand, and a detailed, intensive, in-depth research of data was carried out. However, we highlight the important aspect that the pedagogical experience of this case fostered students to work on problematization, research and interdisciplinary. The studied sample is a class constituted by eight students attending the 3rd semester of the bachelor programme in Engineering. In this PC, two students worked as a pedagogical pair and the others worked individually, depending on their preference.

Govaris (2011) argues that, when implementing PBL as a tool for knowledge consolidation, it is necessary to choose groups with a small number of students. If a significant number of students constitutes the work groups, each of them will only be responsible for a limited part of the task; however, if groups are integrated by two or three elements, they will all be responsible for considering the details, overcoming obstacles and becoming more involved in the correct fulfilment of the task. Table 2 provides a characterizing of the scientific knowledge addressed by these students in other curricular units they had already attended, which may be considered related to the content involved in the PC proposed.

Table 2. Curricular Units attended by Students

semester	curricular unit	related contents and digital materials
1st semester	Calculus 1	Simple integrals to calculate an area on a two-dimensional plane Knowledge of *symbolab* NB: They became acquainted with *desmos* and with the use of an *excel* spreadsheet to format cells, allowing them to expedite numerical calculations.
	Calculus 2	Double integrals to calculate an area on a two-dimensional plane
	Introduction to the study of engineering	Density of a body (Not as a concept of function)
2nd semester	3D Modelling	Technical drawing: modelling with S*olidWorks*
	Technical drawing	Calculation of densities from volume differences (Without resorting to integral calculus)
	Mechanics and waves	Centre of mass (Without resorting to integral calculus)
	Calculus 2	Calculation of centre of mass using double integrals (two-dimensional objects) Use of simple or double integrals to calculate surface areas placed on a two-dimensional frame of reference
3rd semester	Applied Mechanics	Calculation of centroids and centre of mass (Without resorting to integral calculus)

To analyse data, in view of an answer to the research question initially formulated, the following were used as methodological tools: the solutions to the PC, the answers given by students to the questions for reflection set by the lecturer, and a quantitative evaluation rubric of the activity implemented.

Results

Answers to Question 1 of the PC

To be able to start addressing the PC, students realised they would have to begin by geometrically modelling their object, which can be observed in Table 3.

Table 3. Geometric Models involved in the PC

object	identification of the geometric model
Salt container	“Cut” cone (by a horizontal plane)
Candle	Cylinder
Spherical sugar bowl	“Cut” sphere (by a horizontal plane at the bottom)
Metallic bowl	“Cut” paraboloid (by a horizontal plane at the top)
Metallic box	Parallelepiped

Students calculated density by the quotient between the mass and the volume, having realized they could do so, since the density function was constant. Mass was calculated weighing the object with the scales provided. The ruler was used to measure the dimensions of the objects, with a view to making as close as possible, a representation in a three-dimensional Cartesian frame of reference. Volume was calculated using a triple ($V = \iiint dv$) or double ($V = \iint f(x, y) - g(x, y)dxdy$) integral, depending on the geometric shape of the object, as illustrated in Table 4.

Table 4. Calculating the Object’s Volume

geometric model	volume calculus	coordinates used
“cut” cone	double integral	Polar
cylinder	triple integral	Cylindrical
“cut” sphere	triple integral (sphere)	Spherical
	double integral (cut to be subtracted)	Polar
“cut” paraboloid	double integral	Polar
parallelepiped	triple integral	Cartesian

Answers to Question 2 of the PC

Students used the definition of each coordinate of the centre of mass and solve the triple integral in question, with the value of the density obtained in the previous answer, using *symbolab* for the numerical calculation of the integral. Table 5 describes the solution presented, according to the geometrical model considered.

Table 5. Calculating the Object's Centre of Mass

geometric model	calculation of the centre of mass	coordinates used
"Cut" cone	Triple integral, in Cartesian coordinates, including the equation of the cone and conversion to polar coordinates	Cartesian and polar
Cylinder	Triple integral	Cylindrical
"Cut" sphere	Not being able to calculate the integral for the geometric model initially considered, students opted to suppose at this moment that the sphere was whole. They used a triple integral.	Spherical
"Cut" paraboloid	Used the difference between the whole paraboloid and the cut. Used a triple integral, including the paraboloid equation	Cartesian
Parallelepiped	Triple integral	Cartesian

Answers to Question 3 of the PC

Regarding the calculation of the total surface area of the object, students submitted different solution strategies, which can be checked in Table 6.

Table 6. Calculating the Object's Area

geometric model	strategy	notes
"cut" cone	The student resorted to a difference in areas, between the whole cone and the cut, and the formula $A = \iint \sqrt{f_x^2 + f_y^2 + 1}\, dxdy$	Students took care to add the areas of the (circular) base
cylinder	Decomposition of the side surface into two circles and a rectangle and calculation of the area of each using a double integral (Cartesian and polar coordinates).	One of the students who received this model did not submit the calculation of the area
"cut" sphere	Not being able to calculate the integral for the geometric model considered initially, students opted to assume at this moment that the sphere was whole. They used the formula $A = \iint \sqrt{f_x^2 + f_y^2 + 1}\, dxdy$	Students were careful to add the area of the (circular) base
"cut" paraboloid	The student used a difference in areas between the whole paraboloid and the cut, and the formula $A = \iint \sqrt{f_x^2 + f_y^2 + 1}\, dxdy$	The student took care to add the areas of the (circular) bases
parallelepiped	Decomposition of the side surface into three different rectangles and calculation of the area of each of them using a double integral (Cartesian coordinates)	

Answers to Question 4 of the PC

In view of the suggestion made to the students that they must consider an extension of the problem presented in the PC, three of the eight students presented the forward proposals which we organized by categories, as displayed in Table 7.

Table 7. Extensions of PC1

different way to calculate volume	different way to calculate the centre of mass	other applications of integral calculations
"The volume of the solid could be calculated differently: "Filling a measurer with water and checking the volume it indicates; immerse the object and check the volume of the water; subtract the initial volume from the latter one."	"We can use the *CAD Fusion* software to model the object and obtain the coordinates of the centre of mass, the volume and the surface area."	"The use of integrals can be applied to the calculation of distributions of continuous loads on a particular object."
"Geometrically modelling an object in a CAD software programme such as SolidWorks would also make it possible to calculate its volume."		"Triple integrals can also be used to calculate the moment of inertia."

Answers to Reflection Questions

The questions for reflection were answered on *googleforms*, after presentation and discussion session in the classroom. Below we describe the different answers given by the students. On *Q1. If an object were filled half with a substance and half with another substance, what changes would there be in the resolution of the challenge?,* students answered:

> To calculate the centre of mass and the mass density, a simplification was made considering that the object was filled with one single material. If two substances were considered we would have to define whether it would be a homogenous mixture and, if this was the case, the calculation would have to be made in two stages and we would find two density functions." and "To calculate the centre of mass we would need to calculate each one individually and then add the two, since we would consider the object centred at the origin of the frame of reference, whereupon the centre of mass would only vary according to OZ.

Regarding "*Q2. How and why did you decide to place the Cartesian frame of reference in that position relative to the object? What would happen if you placed it differently?"* the students provided the following reflections:

> We decided to place the Cartesian frame of reference so as to achieve a better interpretation of the exercise and modelling of the solid. Had we opted to choose another position in the frame of reference, the integration limits and the whole resolution of the problem would be different.", "I opted to place the object with the base centred at the origin of the XOY plane to facilitate the calculations, with the centre

> of mass only with non-null coordinates, on Z. Had the frame of reference been placed in any other location, which would not have happened." and "Since our object was a cut sphere, we recognized symmetries on the OX and OY axes.

On the reflection proposed by **"***Q3. If this was your case, why did you decide to change coordinates to calculate the integral and did not use Cartesian coordinates?"* students stated their opinions, saying: "Changing the Cartesian coordinates to polar or spheric coordinates enabled us to reach a quicker and easier resolution of the integral, without having to use digital tools such as *Symbolab*." or "As the object is a cylinder, the shift from Cartesian to cylindrical coordinates facilitates the calculation of the integrals."

On **"***Q4. What are your thoughts when you compare the exercises done in class before this challenge with the problem given to you for this challenge?"* students said: "In the end we had a more practical and realistic perception of the concepts learnt in the classroom. This in some way provided the students with tools that in future will help them overcome difficulties, especially when solving problems.", "Although in the case of this challenge the exercises done in class were quite explanatory as regards the applicability of the subject matters taught, with this challenge it was possible to understand the practical difficulties associated with measuring the object and the simplifications we sometimes have to make to achieve actual results. Besides, the main difference involved the fact that we do not work with exercises made and design at the level of calculus neither do we work with actual measurements and values" and "The challenge was a great help to me to understand how the contents applied in the exercises work in real life".

Discussion

In this section, data collected and theories analysed are combined. Observing Table 1, we notice that all the curricular contents that had been aimed to apply were indeed applied, reading the answers in Tables 4, 5 e 6. Regarding the scientific knowledge that was intended to be revealed, the students also met the expectations, which can be concluded when analysing Table 3. We also note that the skills to be developed were also achieved, the significant use of the frame of reference can be seen in the answers to Q2 for reflection, the use of digital tools such as *Symbolab* and the perception of interdisciplinary, namely with the field of Physics, as can be seen in the answers given to the extension of the PC, in Table 7. All students identified the geometric model correctly. In their answers to Q1, students demonstrated having understood the concept of mass density function, as well as the impact it would have in the solution if it was not constant. All the students were able to calculate volume correctly using a double or triple integral. To do this, they used different types of coordinates. In the students answers to Q2 of the reflection set, it becomes clear to the authors of this paper that the choice of position in the three-dimensional frame of reference was a conscious one, considering the position which would most facilitate the calculations to be performed. Analysing the answers to Q2, all the students were able to calculate the coordinates of the centre of mass of their object correctly, despite the different inherent degrees of difficulty. In the case of the students who had to shift from Cartesian to cylindrical coordinates, it becomes apparent, reading their answers to Q3 of the reflection set, that the algebraic calculation of the integral became much easier. When striving to answer Q3, all the students managed to calculate the object's total surface area, having used different strategies.

Three students were able to present extensions to the problem set (see Table 7), on different levels: using *software* they were familiar with from other curricular units to model the object; suggesting the use of triple integrals to calculate the distribution of loads or moment of inertia, in physics contexts, known from other curricular units, thus acknowledging their applicability and usefulness.

Developing the ability to extend the problem has a strong creative component. Mursid et al (2022) realized a study that examined the effect of the blended project-based learning (PjBL) model and creative thinking ability on engineering students drawing learning outcomes. The results showed that the members of an experimental group, comprising students who participated in the blended PjBL model, obtained the highest engineering drawing learning outcomes. The creative thinking ability outcomes of the students taught with this model were higher than those of other group of students who used the ordinary blended learning model. However, those with low thinking abilities taught with the ordinary model exhibited higher learning outcomes than the experimental group. These results show an interaction between the effect of the PjBL model and creative thinking ability on the learning outcomes of engineering students. Therefore, these results support the importance of stimulate creative thinking.

Observing Table 2, we can conclude that this PC fostered the complementarity and interdisciplinarity of scientific knowledge between several curricular units. The concepts of density and centre of mass had already been addressed in *Introduction to the study of Engineering, Technical Drawing,* and *Applied Mechanics*, albeit from different perspectives which did not include integral calculations. Regarding the mathematical modelling proposed in this PC with a geometric model defined by algebraic expressions of functions, the link to computer modelling known by the students from *Solidworks*, in the curricular unit of *3D* Modelling, was acknowledged and identifies. We should recall the criticism levelled by Widjaja et al (2019), regarding the disconnection between the different curricular units, which was eliminated here.

In fact, the idea that solving problems and modelling are intrinsically linked and conduct to find different strategies is corroborated by Yilmaz and Tekin Dede (2016) in a study involving pre-service teachers. They concluded that, in general, considering the problem-solving approaches of the participants, it was thought that more active solution approaches arose due the fact that students realized the solution according to the steps of the modelling process by knowing this process. Based on the findings of the study, it is thought that pre-service teachers present richer approaches not only in the sense of mathematization competency but in the sense of all modelling competencies in general. These richer approaches include a diversity of solutions involving more comprehensive mathematical models gradually during the solution process. In our pedagogical challenge, which involved modelling, students revealed different strategies for solving some of the topics. The PBL methodology applied via the PC proposed aligns with Hydayah et al (2020) as concerns its design model, with the added proposal for an extension to the challenge. Students revealed they were able to work autonomously, as well as solve and overcome any difficulties that emerged, an idea defended by Govaris (2011).

We can also say that the pedagogical model of this experience has characteristics as the one followed by Tanjung et al (2022). In their research, they aimed to determine whether there is an influence of applying the Problem Based Learning (PBL) Model with the Technological, Pedagogical, and Content Knowledge (TPACK) Approach on the historical learning outcomes of high school students of Sultan Iskandar Muda Medan. TPACK consists on material knowledge, pedagogical knowledge, technological knowledge, pedagogical content knowledge, technological and material knowledge, knowledge of technology and pedagogy and knowledge of technology, pedagogy, and materials. They also referred main characteristics of the Discovery Learning Model: explore and solve problems in order to create, combine and generalize knowledge, student-centred and activities to combine new knowledge and existing knowledge. After the pedagogical experience, they concluded that there is a significant difference in learning outcomes, namely the results of experiment class learning that used PBL learning model with TPACK approach were higher than control class learning results that used Discovery Learning Model.

Conclusion

Together, as a group, students reflected on the work done, identifying difficulties and skills developed, and the lecturer recorded their thoughts. Thus, the main difficulties mentioned were: calculating the surface area of the parallelepiped using integrals, calculating the coordinates of the centre of mass of the cut sphere, and calculating the location of the centre of mass by decomposing the structure and subtracting the non-existent part. Regarding the skills developed, students pointed out: work autonomy, development of special visualization compared to what they are used to seeing merely on paper, improvement in the manipulation of coordinates and their conversions.

Students corroborated the opinion of Hydayah et al (2020) when they mentioned the increase in motivation and the expansion of their abilities, by carrying out tasks of this nature. As overall balance, the students reported: "It was quite positive because of the way we were stimulated to use the contents taught in class in an empirical situation."; "The main advantage, as I see it, concerns the practical applicability of the contents of the syllabus, which caused me some difficulties, especially in addressing the problem and how to overcome the issues I was facing.", "I enjoyed doing the challenge, particularly because it was outside my comfort zone.", "As a form of learning and evaluation, it seemed an excellent initiative which draws us outside the usual format of teaching and which, in my opinion, becomes even more effective in grasping and applying knowledge." and "Although it was quite laborious (both for the lecturer and the students), this is an initiative to maintain!". These statements also corroborate the claims of Cuzzuol et al (2018). The answers given to Q4 of the reflection set enabled us to understand that the students believed they had acquired tools that could help them overcome future difficulties, they grasped the need for simplifications which sometimes must be done to actual models, to be able to achieve important results, and acknowledged that their level of awareness regarding the application of the theory to practice, increased after the development of this PC.

As for the research question initially formulated, we can now conclude that, analysing the answers given by the students and the statements quoted, this methodology contributed to foster motivation and efficiency in the significant learning of calculus contents, when applied to these engineering students in the manner described above. The qualitative analysis of the collected data allows us to state that the general and specific goals presented above were fulfilled, once these students of the 2^{nd} year of undergraduate programmes in Aeronautic Sciences and Materials Engineering apply mathematical contents to solve problems, in particular, how they applied geometric modelling to an object and calculate their volume and centre of mass using triple integrals. The main limitations of this study lie in the size of the sample the PC was applied to, although we should add that there was no possible control over any communications between students that may have occurred during their autonomous work, which may have influenced some of the answer they gave. Given the work developed during the present study, we were led to reflect on a way to evaluate quantitatively a PC of this type, in a context of PBL methodology. Thus, we decided to expand this paper in the form of recommendation.

Recommendations

After preparing the theoretical framework we realized that, on one hand, there is no tool for quantitative evaluation of a PBL-based activity, and, on the other hand, that it is important to be able to measure the efficacy of this methodology and the PC. According to Hung (2006), the three main criteria necessary to evaluate a PBL task, following the 3C3R method, are: content, context and connection, as has already been stated. However, also bearing in mind the features of the pedagogical experiment we developed, a quantitative evaluation rubric is proposed and combines these two perspectives. It can be applied to a PC with the characteristics of PBL, in which the lecturer defines:

a) a set of questions for the problem proposed in the PC;
b) an oral presentation to the class;
c) a set of questions for reflection about how was the development of the solution to the problem;
d) a survey on the work done, applied to students.

This rubric will be constructed taking into account its application to any PC with the characteristics mentioned above, and not merely the PC presented in this study, since the two sets of questions, as well as the survey can be adapted to any specific PC that a lecturer may conceive. Since the sets referred to in a) b) and c) have already been elaborated on above, we now present the survey given to the students and which will constitute the final stage of the evaluation:

> "Please, rate your view of the challenge presented, using a scale from 1 (="not at all") to 7 ("a lot"), depending on your opinion.
> D1. Did you have a sufficient theoretical basis to solve the challenge?
> D2. Was the structure of the challenge clear?
> D3. Was the lecturer's support enough?
> D4. In your opinion, were the contents interesting/useful?
> D5. Do you think the challenge was a problem with actual applicability?
> D6. Was the time allocated to solving the challenge adequate?
> D7. Do you think the acquired skills will be useful in other curricular units?
> D8. Was the time given for the presentation and discussion enough?
> D9. Did you like this learning methodology?"

After collecting students' opinions from this questionnaire, it was possible to move on to design the evaluation rubric proposed next, which will enable us to obtain a quantitative evaluation of the task described in this paper. The evaluation rubric we propose and call 2B1Q+O will then be constructed as illustrated in Figure 3.

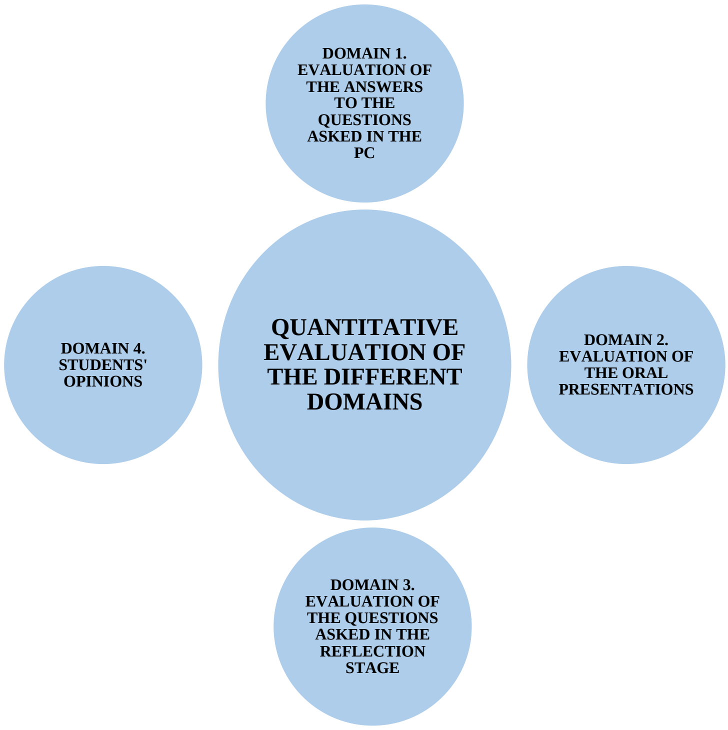

Figure 3. 2B1Q+O Evaluation Rubric

After being applied to the group of students participating in this study, the results obtained can be found in Table 8. The weight of each domain was identical, and for this reason the final quantitative evaluation is an arithmetic average of the evaluation in the four domains. As weaknesses, in this rubric, we identified the subjectivity inherent to the analysis of the results and the format of the PC, which conditioned the application of the evaluation rubric to the definition of the four mentioned domains.

Table 8. Application Evaluation´s Rubric

DOMAIN 1. EVALUATION OF THE ANSWERS TO THE QUESTIONS SET IN THE PC (the lecturer evaluated the answers given by the students to this set of question on a scale of 1 to 20 points, which resulted in a final quantitative score for each student)	Student 1 - 12 Student 2 - 16 Student 3/4 - 16 Student 5- 17 Student 6 - 10 Student 7- 19 Student 8 - 14	quantitative evaluation of domain 1 (arithmetic average) 14.8 points 74%
DOMAIN 2. EVALUATION OF THE ORAL PRESENTATIONS (the lecturer evaluated the students' oral presentations on a scale of 1 to 20 points, defining his own evaluation criteria)	Student 1 - 12 Student 2 - 15 Student 3/4 - 15 Student 5-16 Student 6 - 9 Student 7-18 Student 8-16	quantitative evaluation of domain 2 (arithmetic average) 14.4 points 72%

DOMAIN 3. EVALUATION OF THE ANSWERS TO THE QUESTIONS SET FOR REFLECTION (the lecturer evaluated the answers given by the students to this set of question on a scale of 1 to 20 points, which resulted in a final quantitative score for each student)	(Scale 0-20 points) Student 1 - 15 Student 2 - 15 Student 3/4 - 20 Student 5 - 20 Student 6 - 15 Student 7- 20 Student 8- 15	quantitative evaluation of domain 3 (arithmetic average) 17.1 points 85.5%
DOMAIN 4. STUDENTS' OPINIONS (the students answered the questionnaire via an online form, using an ordinal Likert scale from 1 to 7, subsequently converted to percentage, which resulted in a global evaluation to the 9 questions by each student)	Student 1 – 90.5% Student 2 – 90.5% Student 3 – 93.7% Student 4 – 92.1% Student 5 – 87.3% Student 6 – 95.2% Student 7- 100% Student 8- 90.5%	quantitative evaluation of domain 4 (arithmetic average of the averages) 92.47%
QUANTITATIVE EVALUATION OF THE PEDAGOGICAL CHALLENGE		QUANTITATIVE EVALUATION OF THE DIFFERENT DOMAINS 81%

However, as strengths, this rubric enables each lecturer to formulate the questions they wish to evaluate in each one of the domains indicated, adapting them to their pedagogical and didactical needs, and defining weights to be given to each one domain. For this reason, we recommend the application of this evaluation rubric.

Notes

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