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Epistemological Obstacles of Pre-Service Elementary Teachers in Understanding Fraction

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Abstract

This study aims to identify the epistemological obstacles encountered by pre-service elementary school teachers in understanding the concept of fractions. The study employs a qualitative approach with a phenomenological design to explore students' subjective experiences. The subjects were 73 pre-service elementary teachers who had completed a Basic Mathematics course. Data were collected through a learning obstacle test and semi-structured interviews. The data analysis involved reduction, categorization, and descriptive analysis to identify patterns of epistemological obstacles. The findings revealed that students have a limited understanding of fractions as numbers, visual representations, and fraction operations. Epistemological obstacles include difficulties in understanding fractions as parts of a whole, sets, or points on a number line, as well as common errors in fraction operations such as addition and multiplication. This lack of conceptual understanding is attributed to procedural-focused learning without reinforcing the conceptual meaning. In conclusion, epistemological obstacles in fraction understanding among pre-service elementary teachers highlight the need for more adaptive and concept-based pedagogical approaches. This study recommends the development of more effective curricula and teaching strategies to better prepare future teachers to address the challenges of teaching fractions.

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Introduction

Conceptual understanding is a crucial element in learning, as it supports problem-solving skills, long-term memory retention, and the flexibility of knowledge application. In mathematics, such understanding enables students to apply concepts flexibly while enhancing numerical competence. Relational understanding, as outlined by Skemp (1976), helps students connect various concepts and apply them in different situations. This approach has been proven to promote more meaningful learning, as supported by recent research (Mufanthi, 2020). Expanding on this, Hiebert and Carpenter (1992) highlighted that relational understanding allows students to organize information more effectively, thereby enabling them to solve complex mathematical problems. Furthermore, Kilpatrick, Swafford, and Findell (2001) emphasized that conceptual understanding not only fosters cognitive adaptability but also strengthens critical thinking and creative problem-solving skills. In the context of independent learning, these abilities can be enhanced through approaches such as self-directed learning (Ezaldi, Rizki, & Zulhendri, 2023).

However, prospective teachers often demonstrate limited understanding of fractions, which impacts their teaching effectiveness. Ball (1990) and Li and Smith (2007) noted that prospective teachers frequently struggle with fraction operations, particularly within concrete contexts, leading to misconceptions. This is corroborated by Cramer, Post, and Delmas (2002), who stated that such limitations hinder prospective teachers' ability to explain concepts in ways that are accessible to students. Recent studies have also found that these difficulties are often linked to a lack of experience in applying relational methods in basic mathematics instruction (Tonapa, 2021). Consequently, in the context of fraction learning, students often face challenges in developing a deep understanding, particularly in connecting foundational concepts to their real-world applications.

Mathematics education at the elementary school level plays a fundamental role in establishing students' foundational knowledge, particularly in understanding basic concepts such as fractions. Mastery of fractions is a critical prerequisite for the development of more advanced mathematical thinking, both in academic contexts and everyday life (Siegler, Thompson, & Schneider, 2013). However, various studies reveal that fractions are among the most challenging concepts to comprehend, both for students and prospective teachers (Bruce, Chang, & Flynn, 2013).

These difficulties are often attributed to an excessive focus on procedural tasks without a deep conceptual understanding (Sahin et al., 2020). Additional challenges arise from ineffective teaching methods, such as over-reliance on rote memorization or the use of inappropriate teaching aids (Whiteacre & Nickerson, 2016). Recent research suggests that a strong understanding of fractions requires concept-based approaches, such as the use of number lines and concrete manipulatives, to help students develop more intuitive and flexible comprehension (Faulkenberry & Pierce, 2023).This raises significant concerns regarding the quality of mathematics education at the elementary level, particularly within Elementary School Teacher Education programs, which are responsible for shaping the mathematical competence and understanding of prospective teachers.

The difficulty in understanding the concept of fractions among students can be examined from three perspectives:

epistemological, didactical, and ontogenic. The epistemological perspective highlights that the concept of fractions carries multiple meanings—such as parts of a whole, ratios, or mathematical operators—which require deep understanding from prospective teachers. However, many still experience fundamental misconceptions, such as interpreting fractions as part-whole congruent parts (Charalambous & Pitta-Pantazi, 2007; Jalali, Ikram, & Irawan, 2017).

The didactical perspective focuses on challenges in employing effective teaching strategies, where teachers often fail to utilize visual models that could help students intuitively grasp fractions (Newton, 2008). The ontogenic perspective refers to students' cognitive limitations, as many struggle with abstract thinking, making it difficult for them to comprehend fractions as a complex mathematical concept (Brousseau, 2002). These three factors are interconnected and collectively present significant challenges in teaching the concept of fractions.

Recent research reveals that prospective teachers, including students in Elementary School Teacher Education (PGSD) programs, continue to experience fundamental misconceptions about the concept of fractions, as highlighted by Jalali, Ikram, and Irawan (2017). Charalambous and Pitta-Pantazi (2007) and Clarke, Roche, and Mitchell (2008) emphasize the importance of a flexible understanding of the various meanings of fractions—for instance, as comparisons and ratios—and stress the need for visual models to help address the abstract nature of fractions. Similarly, Tarmizi et al. (2010) and Ni & Zhou (2010) indicate that many prospective teachers lack a deep understanding of fractions, underscoring the need for a holistic and adaptive approach in teacher education to better respond to student needs. Previous studies on misconceptions about fractions among Elementary School Teacher Education programs students often adopt a general perspective, focusing on procedural errors or basic conceptual misunderstandings without distinguishing the underlying causes across specific domains.

In contrast, this study will focus on epistemic obstacles, specifically prospective teachers' understanding of the abstract nature of fractions, such as their interpretation as ratios, part-whole relationships, or numerical values, which are frequently misunderstood. This approach avoids emphasizing didactical (teaching strategies) or ontogenic (developmental stages) aspects, allowing for a more targeted analysis of conceptual difficulties. By delving deeper into these obstacles, the study aims to provide insights that will help design teaching methods tailored to address specific challenges. This approach ultimately leads to concrete recommendations for improving teacher education curricula, particularly in areas related to basic mathematics teaching strategies. In this context, this study aims to identify epistemological obstacles in learning the concept of fractions among Elementary School Teacher Education programs students. A deep understanding of these obstacles is expected to contribute to the development of more effective pedagogical approaches to teaching fractions, particularly at the elementary education level. Furthermore, this study is anticipated to provide guidance in designing curricula and teaching strategies that are more adaptive for Elementary School Teacher Education programs students, preparing them to face the challenges of teaching fractions to their future students.

Epistemological obstacles related to fractions encompass several key indicators that hinder conceptual understanding. First, prospective teachers often struggle to comprehend fractions as numbers, particularly when placing them on a number line or comparing fractional values without equalizing their denominators. Second, there is a gap in connecting visual representations (such as area diagrams or number lines) with the symbolic form of fractions, as visual models are often perceived merely as illustrations rather than tools for understanding. Third, common errors occur in fraction operations, such as overgeneralizing integer rules to fractions without understanding the rationale behind correct operational rules. Additionally, weak understanding of fraction magnitude causes prospective teachers to struggle with estimating the outcomes of fraction operations or grasping the role of fractions in comparison and proportional contexts. These obstacles often arise from a fragmented understanding of the meaning of fractions and a lack of epistemological approaches in teacher education programs (Siegler et al., 2013; Singh & Hiebert, 2019; DeWolf et al., 2015).

Previous research on epistemological obstacles in understanding fractions has identified several significant conceptual challenges. One major obstacle is the limited comprehension of fractions as ratios or comparisons, where many students perceive fractions solely as parts of a whole, making it difficult for them to understand the relationship between the two values within a fraction (Charalambous & Pitta-Pantazi, 2007; Cramer et al., 2002). Additionally, difficulties arise in connecting visual representations of fractions (such as diagrams or number lines) with their symbolic forms, which can impede a deeper understanding of the concept (Siegler, Thompson, & Schneider, 2013). Misconceptions also occur in fraction operations, where students and prospective teachers often fail to grasp the rationale behind operational rules, such as multiplying by the reciprocal in fraction division, and tend to misapply rules meant for whole numbers (DeWolf et al., 2015).

Another challenge lies in applying fractions to real-world contexts, such as price comparisons or measurements, due to a lack of understanding of the flexibility of fractions in various practical situations (Siegler et al., 2013). Finally, there are obstacles in understanding the order and comparison of fractions, particularly when the fractions have different denominators (Siegler et al., 2013). This research highlights the need for a deeper epistemological approach to address these obstacles in fraction learning.

Most existing research tends to focus on understanding fractions from the students' perspective, aiming to identify the conceptual obstacles they encounter in comprehending this subject. However, the understanding of prospective teachers regarding fractions also requires in-depth exploration, as they play a central role in transferring this knowledge to students. More comprehensive research is needed to examine how prospective teachers develop a deeper and more integrative understanding of fractions, including how they connect various representations of fraction concepts, both symbolic and visual. On the other hand, the epistemic challenges faced by prospective teachers are often different from those encountered by students. Prospective teachers must not only understand the concepts in depth but also be capable of effectively communicating and transferring this knowledge to students who may still be in the early stages of understanding.

Theoretical Framework

The theoretical framework of this study employs the concept of learning obstacles, encompassing three primary barriers to understanding the concept of fractions: ontogenic, epistemological, and didactical (Brousseau, 1970). Ontogenic obstacles arise when the instructional material is misaligned with students' mental readiness. These barriers are related to students' cognitive limitations in abstract thinking, which hinder their comprehension of more complex fraction concepts. This leads to difficulties in understanding the material, highlighting a gap between the complexity of the subject matter and the students' level of understanding.

Epistemological obstacles, on the other hand, occur due to the limitations of students' knowledge contexts when first encountering a concept. Students often understand concepts only partially, and when faced with different contexts, they struggle to apply them. This reflects a gap between prior learning experiences and the demands of applying the concept in more diverse contexts. These obstacles arise because fractions have multiple meanings—such as parts of a whole, ratios, or operators—which can confuse prospective teachers who have not yet grasped the flexibility of these meanings (Charalambous & Pitta-Pantazi, 2007).

Didactical obstacles stem from the methods or approaches used by educators. Inappropriate teaching methods can create learning difficulties for students. Didactical barriers emerge when fraction instruction does not utilize visual or concrete models, making it challenging for students to intuitively understand fractions (Clarke, Roche, & Mitchell, 2008). These three obstacles underscore the importance of teaching approaches that are more visual, flexible, and adaptive in enhancing prospective teachers' understanding of the concept of fractions.

Method

This study employs a qualitative approach with a phenomenological design to explore the epistemological obstacles faced by pre-service elementary school teachers in understanding the concept of fractions. Phenomenology was chosen because it enables researchers to deeply comprehend individuals' subjective experiences and identify the essence of those experiences within a specific context (Creswell, 2017; Moustakas, 1994). This approach focuses on "lived experiences," or the experiences directly encountered by the research subjects. In this context, phenomenology is highly effective in uncovering how pre-service elementary school teachers understand and interpret the difficulties they face with fractions.

By examining the students' subjective experiences, this study seeks to identify the epistemological barriers they encounter and how their understanding evolves over time. Furthermore, phenomenology allows researchers to delve deeper into the meanings derived from individual experiences, which cannot be fully captured through quantitative approaches or surface-level analysis. Therefore, the phenomenological approach is particularly relevant for investigating profound epistemological challenges in learning fractions among pre-service elementary school teachers.

The research subjects were pre-service elementary school teachers at a university in Makassar who had completed a Basic Mathematics course. A total of 73 students were given a test, and those who made errors in their answers and could provide in-depth information relevant to the research questions were selected as subjects. The selection process employed a purposive sampling method to ensure that the participants had prior experience and basic understanding of fractions but might still encounter difficulties in comprehensively understanding the material (Creswell, 2017).

In this study, the researcher acted as the key instrument for collecting qualitative data. Data were obtained through a learning obstacle test and interviews with the participants. The learning obstacle test consisted of subject-matter questions designed to uncover the difficulties students experienced in learning fractions. The test items were compiled from various sources (Ciosek and Samborska, 2016; Baturo, 2004; Charalambous and Pitta-Pantazi, 2007; Billstein, Libeskind, and Lott, 1993; Lamon, 2012; Lin et al., 2013) and validated by experts in mathematics and mathematics education. Additionally, a readability test of the questions was conducted with 10 students who had already studied fractions. This step ensured that the learning obstacles identified were not due to poorly understood test items.

Data were collected using several techniques, beginning with a literature review to establish a theoretical foundation on epistemological obstacles in learning fractions. An initial observation was then conducted to directly identify the difficulties faced by students in understanding fractions. The observation focused on aspects such as textbooks, teaching methods, and instructional media used in the learning process. Subsequently, a written test was administered, targeting key aspects such as part-whole understanding, fraction operations, and the interpretation of fractions in specific contexts, to uncover potential obstacles. Following the test, semi-structured interviews were conducted with selected subjects to explore their experiences and perceptions regarding these difficulties in greater depth.

In this study, data were analyzed using the techniques outlined by Miles and Huberman (1984), beginning with data reduction through coding to identify the main themes of epistemological obstacles. The data were then categorized using metamatrices to display patterns of obstacles among students. Descriptive analysis was conducted to provide a detailed characterization of the epistemological obstacles encountered. Finally, a causal network was constructed to identify the key factors contributing to these obstacles and to formulate recommendations for more effective teaching strategies.

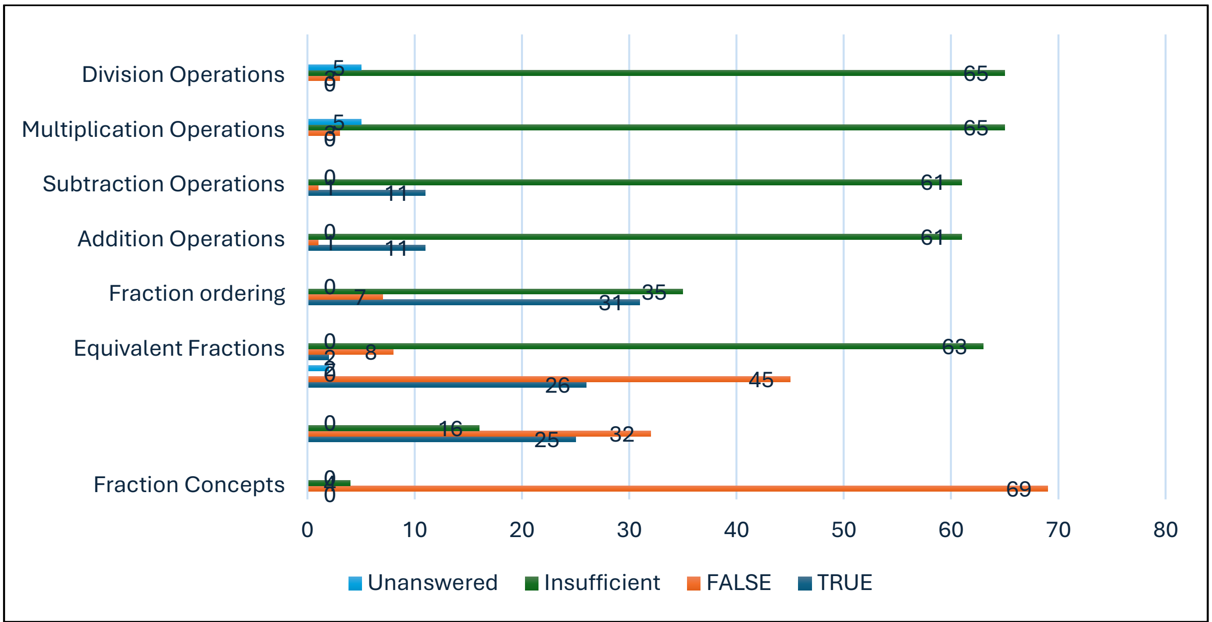


Figure 1. Test Results on Learning Obstacles in Fraction Concepts

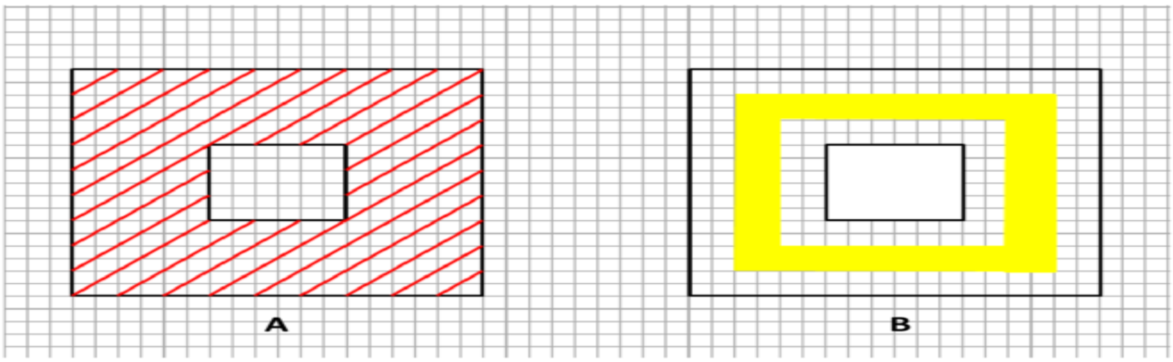
The analytical framework used to examine students' knowledge of fractions is based on the perspective of Lin et al. (2013). According to their view, prospective teachers must understand fractions through both procedural and conceptual knowledge. Additionally, the fraction content they need to master includes: fraction concepts, equivalent fractions, the ordering of fractions, and operations with fractions, which encompass addition, subtraction, multiplication, and division. This framework also informed the researcher in designing the learning obstacle test. The responses provided by the students are illustrated in Figure 1.

Results and Discussion

Student Knowledge of Fraction Concepts

The first section of the analysis is derived from items 1, 2, and 3 of the learning obstacle test, located in rows 1 to 3 from the bottom of Figure 1. These three items focus on the content of fraction concepts, including the part-whole model, the part-group model, and the number line (Bell, A. W., Castello, & Kucheman, 1983). The purpose of item 1 is to reveal students' understanding of the concept of fractions as part of a whole (part-whole). Figure 2 illustrates several student responses based on their answers to item 1.

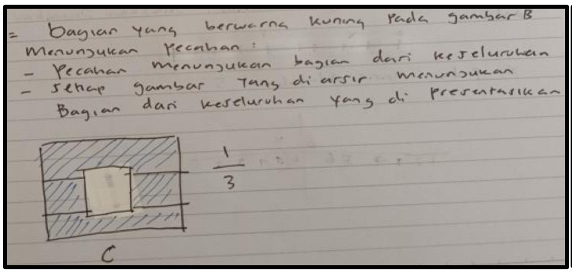
Question 1

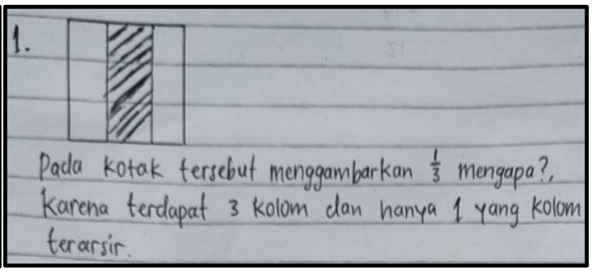


If the shaded area in **Figure A** represents 1 unit, does the yellow-shaded portion in **Figure B** represent the fraction $\frac{1}{3}$? If **yes**, explain how you arrived at your answer! If **no**, draw a **Figure C** that correctly represents $\frac{1}{3}$! (Ciosek and Samborska, 2016)

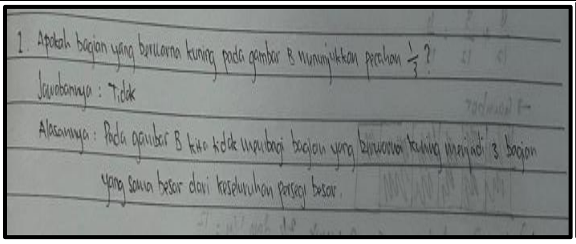
Student Responses in Indonesian

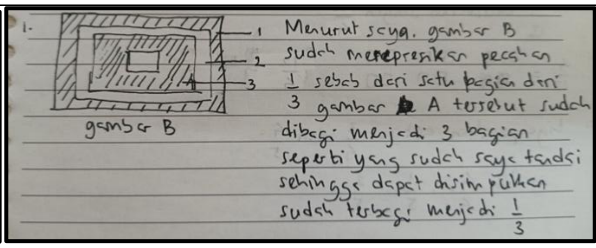
(a) (c)





(b) (d)





Student Responses in English

(a) The part that is yellow in image B represents a fraction:
- A fraction represents a part of the whole.
- The shaded area in the image represents the part of the whole being presented.

See the picture from Figure 2(a) showing the student's response in Indonesian

(b) **Does the yellow part in image B represent the fraction $\frac{1}{3}$?**

Answer: No

Reason: In image B, the yellow part does not represent $\frac{1}{3}$ of the larger whole part of the big rectangle.

(c) Does the box represent $\frac{1}{3}$? Why? Because there are 3 columns, and only 1 column is shaded. See the picture from Figure 2(c) showing the student's response in Indonesian

(d) In my opinion, picture B already represents the fraction $\frac{1}{3}$ of a whole rectangle. Picture A has been divided into 3 parts, as I explained earlier. Thus, it can be concluded that it has been divided into $\frac{1}{3}$. See the picture from Figure 2(d) showing the student's response in Indonesian

Figure 2. Student's Answer to Question 1

Student responses to question 1 were quite diverse. Based on Figure 2, only 4 out of 73 students provided the correct answer. This means that the majority of students gave the wrong answer, namely "no." However, the correct answer cannot be considered valid without considering the reasoning provided by the students. Therefore, the researcher classified the responses to question 1 into two categories as follows:

Answer: Yes, it is $\frac{1}{3}$

Four students answered, "Yes, it is $\frac{1}{3}$." This can be seen in Figure 2(d). The researcher conducted further interviews regarding this answer. Below is an excerpt from the student's statement:

Researcher (P): What do you understand about this question? Can you explain it to me?
Student 4 (S4): Yes, sir. Here, it says (reading the question), so I answered yes, because there is yellow there, right? There are already three parts, so the first is 1, then 1 for the outer part, then 2 for the yellow, and then 3 for the inner part again. So, I represented it as $\frac{1}{3}$.

All students who answered "Yes, it is $\frac{1}{3}$" provided the same reasoning: the yellow part represents $\frac{1}{3}$ because Figure B in question 1 is divided into three parts, and one of them is shaded yellow (Figure 2(d)). This reasoning is clearly incorrect because the three parts are not equal in size, and thus the explanation fails to fulfill the required answer. This statement also contradicts the concept of fractions as expressed by Karim (1998), which states that a fraction is the comparison of equal parts to the whole, also known as the concept of fair sharing. Out of 73 students, none were able to provide an answer accompanied by correct reasoning. This is surprising because the question is relatively simple, as it was adapted from a 4th-grade textbook used by students in Poland (Ciosek, M., Legutko, M., Turnau, S., & Urbanska, 2005).

Based on these findings, the researcher concluded insights regarding students' understanding of the concept of fractions as part of a region (part-whole). First, students have not fully grasped the concept of fractions as a part of a region; they only partially understand the concept (see S4's response). They understand that to obtain $\frac{1}{n}$ of a given whole region, the entire region must be divided into n equal parts (Ciosek & Samborska, 2016). However, they do not apply this knowledge when solving problems. This indicates that their knowledge is not yet automatic. Below is an excerpt from S4's response:

P: For example, let's say I or you break a glass onto the floor. If the glass shatters on the floor, there are several pieces, right? Can the shards be represented as a fraction or a fractional concept? After all, the sizes of the pieces are different, aren't they? It's unlikely they'll all be the same size, right?

S4: Yes, sir. In my opinion, no.

P: What is your reason for saying no?

S4: Because, well, maybe it's because, for instance, if some of the pieces are small, like some shards are tiny, sir, and some are really big, and some might even be completely shattered into dust.

Second, although some students provided the correct answer, the reasoning they gave was still inaccurate. The reasoning was not rational and revealed uncertainty in their responses, as seen when the researcher posed a different question with the same concept (see S4's response). S4 initially answered according to the correct concept but ultimately contradicted their response to question 1. Third, students were unable to fully interpret the information provided in the question. In question 1, gridlines were provided, which they could have used to calculate the area representing $\frac{1}{3}$, but they failed to utilize them. This further highlights the students' lack of conceptual understanding of fractions.

Answer: No, it is not $\frac{1}{3}$

From Figure 1, it was found that 69 students answered: "$\frac{1}{3}$." Here are some excerpts from the students' statements:

Student 1 (S1): I divided the box into 3 parts. For point C, we are asked to make it, so if the student doesn't find $\frac{1}{3}$ *, they should draw it themselves. So, I made it in part C.*

P: So, you agree that Figure B does not represent $\frac{1}{3}$*?*

S1: Yes.

A similar statement was made by another student, here is the excerpt from their statement:

Student 2 (S2): At first, the reason for my answer is no, because I didn't really pay attention to the middle square *that was not shaded. I suddenly thought, 'Oh, it doesn't seem right,' so I immediately said no.*

Student 3 (S3): I came to this conclusion because the problem asked us to draw $\frac{1}{3}$ in one box, so I divided it into 3 parts and shaded it like that.

The students who provided these answers largely did so because they did not fully understand the question. This can be seen in the interview excerpts from S1, S2, and S3 in Figures 2(a), 2(b), and 2(c). Moreover, it was not only these students who misunderstood Question 1—other students who gave the same answer also provided similar reasoning, as they believed that Figure B did not accurately represent $\frac{1}{3}$. As a result, they created Figure C, which they thought properly represented $\frac{1}{3}$. This indicates that many students might have misunderstood the structure of the question or misinterpreted the visual representation, leading them to propose an alternative solution (Figure C) based on their own interpretation of how $\frac{1}{3}$ should be represented.

Based on this, the author concludes that the students do not understand fractions as a unified whole (Lamon, 2012). They made an error in interpreting the question because they did not pay attention to the fact that the shaded area in Figure A was considered as one whole unit, meaning that Figure A and Figure B were actually connected. They assumed that Figure A was unimportant and thus focused their attention solely on Figure B.

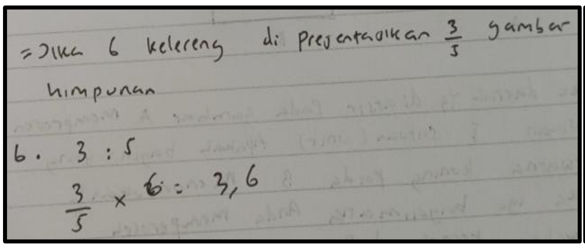
The following section discusses Question 2, which aims to reveal students' understanding of fractions as part of a set (part-set). Figure 3 presents some student answers for Question 2.

Question 2

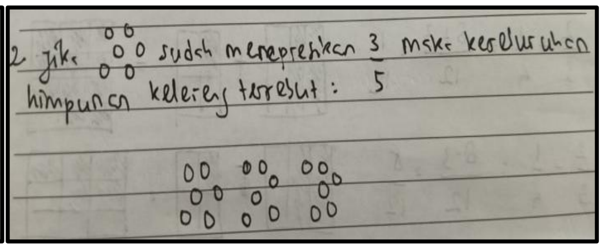
If  represent $\frac{3}{5}$ of the set of marbles, draw the entire set of marbles! (Baturo, 2004)

Student Responses in Indonesian

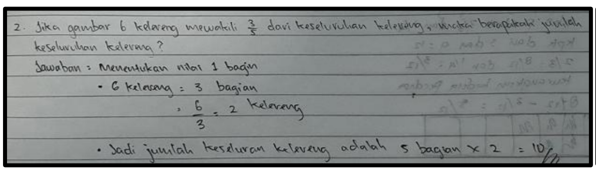
(a)



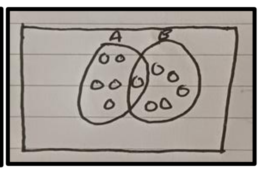
(c)



(b)



(d)



Student Responses in English

(a) If 6 marbles represent $\frac{3}{5}$, draw the set.

6 . 3 : 5

$\frac{3}{5} \times 6 = 3{,}6$

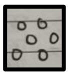
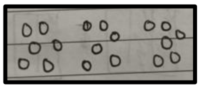
(b) **2.** If the picture of a wheel represents $\frac{3}{5}$ of the total number of wheels, how many wheels are there in total?

Guide: Calculate the value of 1 part.

6 wheels = 3 parts.

$\frac{6}{3} = 2$ wheels

So, the total number of wheels is: 5 parts × 2 =10

(c) If  already represents $\frac{3}{5}$ then the total number of the set of circles is: 

(d) See the picture from Figure 3(d) showing the student's response in Indonesian

Figure 3. Student's Answer to Question 2

Similar to Question 1, the answers provided by the students for Question 2 were varied. Based on Figure 1, 41 students answered correctly, but 16 of them provided incorrect reasoning, and the remaining students gave wrong answers. In Figures 3(c) and 3(d), the respondents simply represented $\frac{3}{5}$ in visual form, which did not address the actual question in Question 2. This is further supported by the following interview excerpts:

S4: I did that because I thought, oh, this represents $\frac{3}{5}$ *, so since I didn't fully understand what a set is, I just drew three images to represent that, that's how I understood it, sir.*

P: Okay, so you drew it three times in Figure 6, right?

S4: Yes.

P: Now, what was going through your mind when you did that? Why did you draw it like that? Why did you think this was the answer to Question 2? It's okay, just say what was in your mind, no need to be shy.

Student 5 (S5): I didn't quite understand, sir.

Based on the interview excerpt from **S5**, the student answered this way because they did not understand the meaning of Question 2. Meanwhile, **S4** explained that, because the question mentioned the fraction $\frac{3}{5}$, they represented 18 marbles by multiplying the 6 marbles that were already present by 3. However, the intent of Question 2 was for the students to draw the entire set of marbles, not to multiply the existing marbles to represent the fraction.

In Figure 3(b), the student actually began with the correct step by writing down the number 10. However, after the interview, it became clear that the student did not understand the intent of Question 2. Here is the excerpt from the interview:

S2: Actually, sir, the important thing was to finish it yesterday, so I worked on it at night, and just took the easy way. I wasn't sure whether it was correct or not. I thought, 'Okay, 3 means 3 parts out of 5, right, sir?

P: Yes.

S2: So, I just divided it, because first I needed to know how many marbles there were, so I just said, 'Oh, I'll divide 6 by 3.' Then 1 part would have 2 marbles, but since there are 5 parts, I just did 5 times 2, which equals 10.

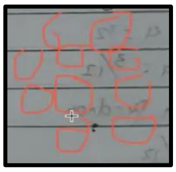
According to S2, the steps they took were procedural, but they did not understand the relationship between $\frac{3}{5}$ and the set of marbles asked about in the question. Even when S2 was asked to represent the result in a visual form as requested in the question, they only drew 10 circles without shading to represent the 6 marbles from the total of 10 marbles in the entire set. This further supports the assumption that S2 did not understand the task in Question 2. Here is an excerpt from the interview:

P: Now, try to draw the marbles as you understand them. How would you represent them?

S2: How should I do it, sir? I'll make circles, is that okay?

P: Yes, that's fine.

S2:



P: Is that it? Nothing else you'd like to add?

S2: I think that's it, sir.

In Figure 3(a), the student provided an answer to Question 2 in a procedural form that resulted in a decimal number. After the interview, the following statement was made:

S1: Maybe I can explain here. There are 6 marbles, and to represent $\frac{3}{5}$, I just did it like this, sir: 3 divided by 5, then I multiplied 3 by 6 to get 18, then divided by 5, so the answer is 3.6.

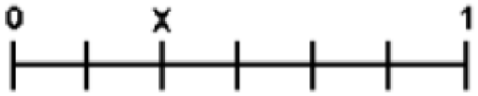
Based on the excerpt above, it is clear that S1 wrote the answer in decimal form because the 6 marbles depicted in the problem were meant to represent $\frac{3}{5}$. They then proceeded to multiply $\frac{3}{5}$ by 6, which resulted in 3.6.

Through this information, the researcher concludes the students' understanding of fractions as part of a set (**part-set**). First, the students do not fully understand the concept of fractions as part of a set, because when the researcher asked follow-up questions regarding their understanding of the same concept, they were able to answer correctly. Second, the students were only able to solve problems that they had previously encountered, even though the concepts involved in the questions were ones they had already learned. This suggests that the students had only received the concepts from their instructors without engaging in meaningful learning processes. This aligns with Ausubel's (1963) view that learning by receiving concepts merely leads students to memorize the concepts provided, without being able to implement them effectively.

The next section discusses Question 3**,** which aims to explore the students' understanding of fractions as a number line. This question is relatively easy, but still, many students made mistakes when attempting to solve it. Based on Figure 1**,** 26 students answered the question correctly, while 45 students gave incorrect answers, and 2 students did not answer the question, citing that they didn't know. Figure 4 shows several excerpts from the students' responses to Question 3.

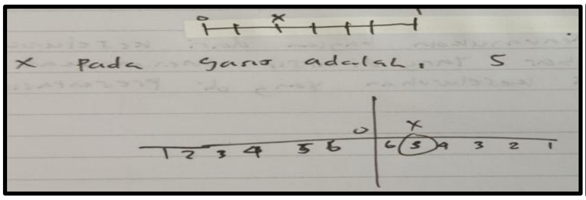
Question 3
What fraction is represented by x on the number line below? Explain your answer! (Charalambous and Pitta-

Pantazi, 2007)

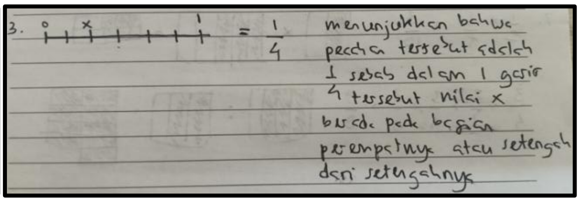


Student Responses in Indonesian

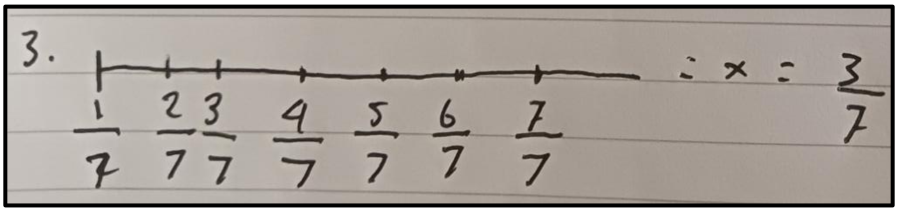
(a)



(c)



(b)



Student Responses in English

(a) x on the number line is: 5. See the picture from Figure 4(a) showing the student's response in Indonesian

(b) See the picture from Figure 4(b) showing the student's response in Indonesian

(c) It shows that the fraction represents one section within 1 unit. $\frac{1}{4}$ is worth x, divided into four equal parts, or half of half. See the picture from Figure 4(c) showing the student's response in Indonesian

Figure 4. Student's Answer to Question 3

As with the previous questions, the answers provided by the students for this question were varied, and the majority of students gave incorrect answers. Based on Figure 1, 26 students provided the correct answer, while 45 students answered incorrectly, and 2 students did not respond. Figure 4 presents the students' responses, some of which show incorrect placements of the fraction on the number line. Here is an interview excerpt from a student whose answer corresponds to Figure 4(a):

S1: Maybe, sir, the one I did is like before, with the principle—what's that? The one next to x, and then y above it, then I fill it with numbers.

P: So, you filled it with numbers. Why did you fill it in as 1, 2, 3, 4, 5, 6, then draw a line?

S1: Actually, sir, I was in a hurry because there was little time left. I should have started with 1, 2, 3, 4, 5, 6, and then next to it, I should have started again from 1.

P: Okay, so if you start from 1 again next to it, does that mean x's value will change, or will it stay the same?

S1: It might stay the same, sir.

P: Alright, if we look at this problem, we see that the lowest value is 0 and the highest value is 1. Now, earlier you got x's value as 5, right? Does that seem correct?

S1: No, sir.

P: Would you like to change your answer, or should it stay 5?

S1: I want to change it.
P: So, what would you change your answer to after hearing my explanation?
S1: Hmm, the smallest value is 0, and the highest value is 1.
P: Yes, that's how it looks in the figure. If you want to change it, what would you change x's value to?
S1: Maybe 0.
P: So, you would change the value of x to 0? Why would it be 0?
S1: I divide the smallest and largest numbers, and then I multiply, sir. If I multiply, 0 times 1 is still 0. Then, 0 divided by 1 is still 0.

From the interview excerpt with S1, it can be concluded that S1 answered x=5x = 5x=5 because they assumed that after the number 1 on the number line, the numbers would continue in sequence, so S1 proceeded with 2, 3, and so on. This indicates that S1 did not fully understand the fundamental concept of the number line, particularly regarding how the distances between numbers are divided. Even when the researcher provided further explanation regarding question 3, S1 changed their answer to 0. However, this change was also incorrect. S1 changed their answer to 0 with the reasoning that 0 multiplied by 1 equals 0, and 0 divided by 1 is still 0. This suggests that S1 attempted to apply mathematical logic that was not appropriate for the context of the question, where the partitioning of space between 0 and 1 on the number line should be proportional rather than based on simple multiplication or division.

This mistake shows a lack of understanding of the concept of the number line, where each point on the line represents a specific fraction, not just a linear counting of numbers. In this case, S1 was unable to accurately determine the position of a number on the number line, which led to an incorrect answer. This analysis underscores the importance of a deeper understanding of the number line concept and how to use it to represent fractions. In this context, students need to be trained to grasp the position of fractions on the number line in a more intuitive and meaningful way, rather than relying solely on rigid mathematical procedures.

In contrast to the reasoning provided by S4, which is shown in Figure 4(c), the following is the excerpt from the interview:
S4: Well, from 0, there's 1 whole part, and then we divide it. If we divide it by $\frac{1}{2}$, it's in the middle near x (pointing at the image in S4's answer), near x on the right side, so it becomes $\frac{1}{2}$ if x is there. Then...
Researcher: So you mean x here? (pointing at the image in S4's answer)
S4: Yes, and since what's being asked is the part where x is, it's already divided into $\frac{1}{2}$. If we divide $\frac{1}{2}$ again, it will be $\frac{1}{4}$, that's how I see it.

According to S4, the answer was given because, in the number line, there is a boundary between 0 and 1. S4 explained that when the position of x, which is being asked in the question, is moved one step to the right, the answer would be $\frac{1}{2}$. After that, S4 concluded that the value of x is $\frac{1}{4}$ because they divided $\frac{1}{2}$ by 2, which resulted in $\frac{1}{4}$. This answer is clearly incorrect because the position of x is not accurately placed in the middle between 0 and

$\frac{1}{2}$. While S4 made a good initial step in considering the divisions on the number line, they did not pay close enough attention to the actual position of x in the question, leading them to draw an incorrect conclusion.

S5's answer in Figure 4(b) is actually interesting because it differs significantly from the answers provided by S1 and S4. Here is an excerpt from the interview:

S5: Well, Sir, the line in the question is divided into 7 parts, so I wrote the numbers 1 to 7.

S5 answered with $\frac{3}{7}$, because the number line is divided into 7 parts, and thus, S5 labeled each section from $\frac{1}{7}$ to $\frac{7}{7}$. However, S5 did not pay attention to the boundaries given in the question, specifically the 0 and 1 points. As a result, the answer provided by S5 is incorrect. On the other hand, S7 gave an answer similar to S5 after being given guidance from the question. However, S7 provided a unique statement during the interview. Here's the excerpt:

P: So, is it correct that 0 equals $\frac{1}{7}$?

Student 7 (S7): Yes.

S7 stated that 0 is equal to $\frac{1}{7}$. This reinforces the suspicion that S7 has not yet been able to connect the knowledge they have learned previously.

Based on this information, the researcher concludes the following regarding the students' understanding of fractions as a number line. First, the students were unable to provide a correct answer because they had not yet understood the positioning of numbers on the number line. Second, students still struggled to comprehend fractions as a number line, even when provided with guidance. Third, students faced difficulty in connecting the knowledge they had previously learned. According to Becker & Shimada (1997), a learner must be able to combine three elements: the new knowledge they acquire, their prior knowledge, skills, and their mathematical thinking approach.

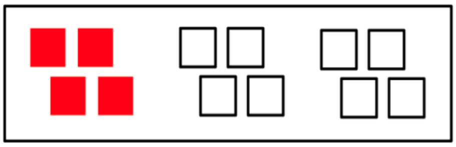
The barriers faced by students in understanding fraction concepts were identified from their responses to questions 1, 2, and 3. Students' understanding of fraction concepts is still quite limited. They have not fully grasped the concept of fractions as part of a whole (part-whole), as part of a set (part-set), fractions as a unified entity (unity), and fractions on a number line. Many of them may have heard or are somewhat familiar with these concepts, but they have not understood them well because they are unable to apply these concepts to the problems presented. This indicates that students are experiencing epistemological obstacles, as their understanding of fractions is still quite limited.

Student Knowledge on Equivalent Fractions

Students' understanding of equivalent fractions can be seen in question 4 or learning obstacle number 4. This question aims to assess how students can solve and explain problems related to equivalent fractions. In Figure 5, several student responses to question 4 are presented.

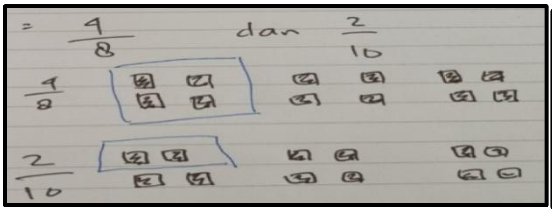
Question 4

Write two equivalent fractions for the following image! Provide your answer with a graphical representation! (Billstein, Libeskind, and Lott, 1993)

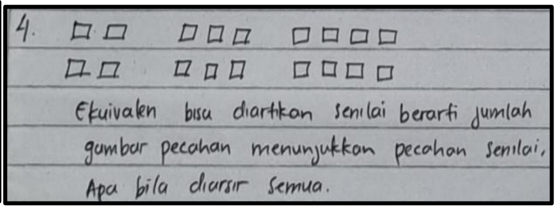


Student Responses in Indonesian

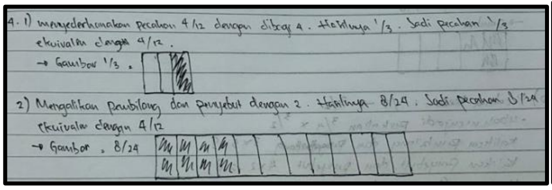
(a)



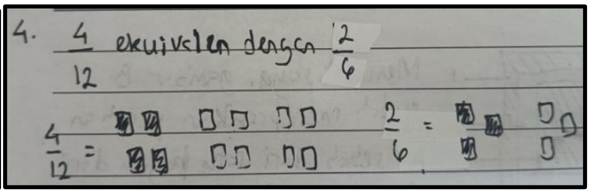
(c)



(b)



(d)



Student Responses in English

(a) $\frac{4}{8}$ and $\frac{2}{10}$. See the picture from Figure 5(a) showing the student's response in Indonesian

(b) 1) Simplifying the fraction $\frac{4}{12}$ by dividing it by 4. The result is $\frac{1}{3}$. Therefore, the fraction $\frac{4}{12}$ is equivalent to $\frac{1}{3}$.

- Illustration: $\frac{1}{3}$. See the picture from Figure 5(b) showing the student's response in Indonesian

2) Explaining the division and simplification of $\frac{8}{24}$ by dividing it by 2. The result is $\frac{4}{12}$. Therefore, the fraction $\frac{8}{24}$ is equivalent to $\frac{4}{12}$.

- Illustration: $\frac{8}{24}$. See the picture from Figure 5(b) showing the student's response in Indonesian

(c) Equivalent can be interpreted as having the same value, meaning the total number of fraction diagrams represents fractions of equal value if all the sections are shaded. See the picture from Figure 5(c) showing the student's response in Indonesian

(d) $\frac{4}{12}$ is equivalent to $\frac{2}{6}$.

Illustration:

$\frac{4}{12}$ = (shaded boxes representing $\frac{4}{12}$). See the picture from Figure 5(d) showing the student's response in Indonesian

$\frac{2}{6}$ = (shaded boxes representing $\frac{2}{6}$). See the picture from Figure 5(d) showing the student's response in Indonesian

Figure 5. Student's Answer to Question 4

Based on Figure 1, only 2 students answered correctly, while 63 students answered correctly but their explanations were incomplete, and 8 others gave incorrect answers. Therefore, the researcher conducted further interviews regarding the difficulties students encountered in answering Question 4. In Figures 5 (b) and (d), the answers from S2 and S4 were actually correct but not fully complete. Below are excerpts from the interviews.

S2: $\frac{1}{3}$ is the same as $\frac{8}{24}$.

P: How did you come to that answer?

S2: Because the question had $\frac{4}{12}$.

P: Where does $\frac{4}{12}$ come from? Let's look at the question.

S2: It's from the image, sir, where the red part is 4, and the total is 12.

P: So why $\frac{1}{3}$?

S2: Because when simplified, $\frac{4}{12}$ becomes $\frac{1}{3}$.

P: How do you simplify it?

S2: You divide both the numerator and denominator.

P: By what number?

S2: By 4.

P: Why did you think of it that way?

S2: Because that's usually what we do, sir.

The same explanation was also given by S4, as seen in the following excerpt:

S4: That's how I did it, sir, I divided it.

P: Divided by what?

S4: I divided by 2.

P: Which part did you divide by 2?

S4: I divided the 4 by 2, which gives 2, and 12 by 2, which gives 6.

P: But in the question, we can see that it asks for two fractions.

S4: Oh, I didn't pay attention to that, sir. I was in a hurry.

P: Two fractions, but...

S4: I only wrote one.

P: If you were to write another fraction, what would it be?

S4: Maybe $\frac{1}{3}$ again.

P: Where does that $\frac{1}{3}$ come from?

S4: I just divided it again by 2. So $\frac{2}{6}$ divided by 2.

From the interview excerpts above, it is evident that S2 and S4 understood the purpose of the question, but they did not provide the correct graphical representation according to Question 4. Instead, they created new images, as seen in Figure 5. This indicates that they had difficulty connecting the provided graphical representation in the question with their answers. Interestingly, the responses from S1 and S3 in Figures 5 (a) and (c) were different. Below is the interview excerpt from S1.

S1: From the picture, sir, because it says to write two equivalent fractions based on the image, and in the image, *there are four shaded parts, and the rest are unshaded.*

P: So, the unshaded parts are not counted, right? That means there are four shaded parts. How many unshaded parts are there?

S1: There are eight.

P: So, you wrote $\frac{4}{8}$*, meaning 4 parts are shaded and 8 parts are unshaded, correct?*

S1: Yes.

P: And what about $\frac{2}{10}$*?*

S1: For $\frac{2}{10}$*, I took the two shaded parts and left the others unshaded.*

P: Oh, so you took two shaded parts and left the others unshaded, which means the total is 10, right? 8 + 2 = 10, so it's $\frac{2}{10}$*.*

According to S1's understanding, the image in the question contains four shaded red parts and eight unshaded parts. S1 then shaded an additional two parts while leaving the rest unshaded. This led S1 to provide the response shown in Figure 5(a). Furthermore, S1 did not grasp the concept of equivalence. The interview excerpt with S3 is as follows:

S3: Perhaps it's different from what the question was asking and different from the previous answers.

P: But if you were given the chance to revise your answer, what would you write? Or would you leave it the same as this?

S3: If I could revise it, I would ask for help to be taught first.

P: Okay. Does that mean you really don't know how to answer the question?

S3: I don't know.

P: Okay, you don't know at all?

S3: Yes.

S3 admitted that the answer given in Figure 5(c) was different from the question's instructions, but they were unable to provide any justification. Even when the researcher gave S3 an opportunity to review the question and try answering again, S3 still did not know how to respond.

Based on the students' answers and interviews, it can be concluded that the causes of students' mistakes in answering Question 4 are as follows: First, the students' limited understanding of fraction concepts led to their inability to comprehend the problem being asked. The content required to solve Question 4 involves two concepts: fractions as parts of a set and equivalent fractions. However, very few students demonstrated an understanding of fractions as parts of a set. They only recognized the concept of fractions as parts of a region, yet they were unable to implement this concept in the question. This is evident from the fact that none of the students answered Question 1 correctly. Since the concept of fractions as parts of a set is new to most students, they were unable to answer Question 4 correctly.

Second, the students' limited understanding of terminology. Some students did not know the meaning of the word "equivalent." The following is an excerpt from the interview:

P: Do you know what "equivalent" means?

S5: I don't know, sir.

Third, the lack of diverse problem sets provided to students. Many students failed to answer the question correctly because they had never encountered similar problems before, even though the solutions used basic concepts. Fourth, the students' lack of familiarity with the concept of unit fractions. This is a significant issue, as the concept of unit fractions is fundamental and should have been introduced at the beginning of their fraction learning. Lamon (2012) emphasizes that a major conceptual problem contributing to students' difficulties in understanding fractions is related to the "unit of fractions." In fractions, a single unit can consist of more than one object, and this concept is often overlooked by students. Many of them assume that a unit fraction refers only to a single object. Thus, the primary reason many students failed to correctly answer Question 4 regarding equivalent fractions is that their understanding of basic fraction concepts remains incomplete.

Additionally, Question 2 revealed an epistemological obstacle due to the students' limited understanding of context. They did not grasp the concept of fractions as parts of a set (part-set), which led to errors in the initial step of interpreting the fraction value from the image in Question 4. Consequently, the students' answers were incorrect.

Students' Knowledge of Fraction Ordering

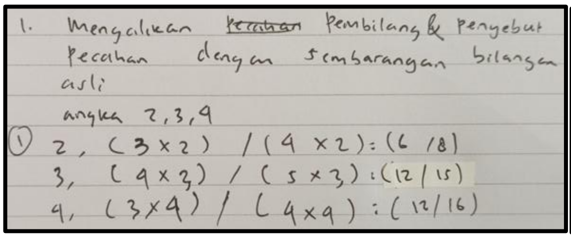
Students' understanding of the properties of fraction ordering can be observed through the test question on learning obstacle number 5. This question aims to assess how students can solve and explain problems related to fraction ordering. Figure 6 presents several student responses to Question 5.

Question 5

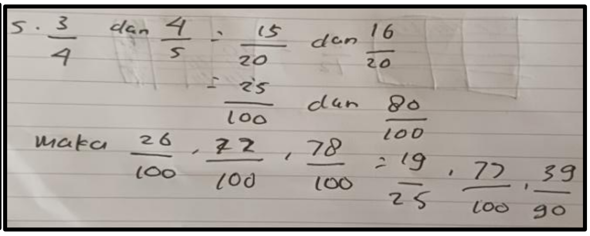
Write three fractions that lie between $\frac{3}{4}$ and $\frac{4}{5}$, and explain how you arrived at your answer! (Lamon, 2012)

Student Responses in Indonesian

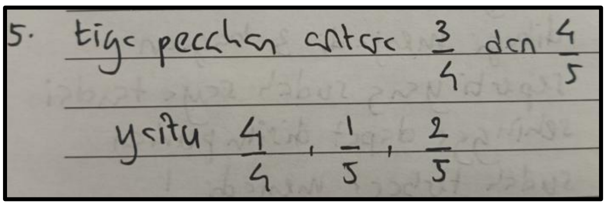
(a)



(c)



(b)



Student Responses in English		
(a) 1. Multiplying the numerator and denominator of a fraction by arbitrary whole numbers:		
Numbers: 2, 3, 4		
2, (3×2)/(4×2)	=	(6/8)
3, (4×3)/(5×3)=		(12/15)
4, (3×4)/(4×4)=(12/16)		
(b) Three fractions between $\frac{3}{4}$ and $\frac{4}{5}$ are: $\frac{4}{4}, \frac{1}{5}, \frac{2}{5}$		
(c) $\frac{3}{4}$ and $\frac{4}{5}$: $\frac{15}{20}$ and $\frac{16}{20}$		
: $\frac{25}{100}$ and $\frac{80}{100}$		
Thus, fractions between them are: $\frac{26}{100}, \frac{72}{100}, \frac{78}{100} = \frac{19}{25}, \frac{77}{100}, \frac{39}{90}$		

Figure 6. Student's Answer to Question 5

Based on Figure 1, 31 students answered correctly, while 35 provided correct answers but with incomplete reasoning, and 7 answered incorrectly. Therefore, the researcher conducted further interviews to explore the challenges students faced when answering Question 5. The excerpt from S1's response in Figure 6(a) is particularly unique, as it indicates that they multiplied the numerator and denominator of the fractions by arbitrary natural numbers. Below is the interview excerpt:

P: So, you understand the question, right? Let's look at your answer here. I see you multiplied the numerator and denominator of the fraction by arbitrary natural numbers. Do you know what natural numbers are?

S1: Natural numbers, yeah, numbers like 1, 2, 3, 4, …

P: Okay, does zero count as a natural number?

S1: Yes, it does.

P: Alright. Why did you multiply by natural numbers? Here you directly arrived at fractions like $\frac{6}{8}, \frac{12}{15}, \frac{12}{16}$. *Why did you think to multiply by natural numbers? What made you decide this approach to answer the question?*

S1: Maybe it's because of the question itself, sir. It asked for three fractions.

P: So, to get three fractions, you simply multiplied by arbitrary numbers, right?

S1: Yes, as long as the values are between $\frac{3}{4}$ and $\frac{4}{5}$, *so the numbers don't exceed* $\frac{4}{5}$.

S1 explained that they multiplied by arbitrary numbers to obtain the three fractions requested in the question, ensuring that the results did not exceed $\frac{4}{5}$. Interestingly, when the researcher asked about which numbers are considered natural numbers, S1 stated that 0 is included as a natural number, which is incorrect. This indicates a misunderstanding of both the question and the concept of natural numbers. The underlying reason for S1's response is their lack of understanding of Question 5 and insufficient familiarity with different types of numbers, specifically natural numbers. On the other hand, S4 provided a different approach, as seen in Figure 6(b). The interview excerpt is as follows:

S4: It's... what is it, sir? Feeling.

P: Answered using feeling, okay, good.

S4: Using feeling, it's just a habit, sir. We know it's usually correct, right? If we don't know, we just use feeling.

S4's response indicates a reliance on intuition or "feeling" due to a lack of procedural knowledge or understanding of the problem. This highlights a significant gap in conceptual and procedural knowledge, as they default to guesswork when faced with uncertainty. A different response came from S7, whose interview excerpt can be seen in Figure 6(c):

S7: But the question is $\frac{3}{4}$ and $\frac{4}{5}$, so here $\frac{15}{20}$, uh, 5 × 3 = 15, whereas where does the 20 come from? Oh, 4×4.

The interview reveals that S7 also guessed the answer to Question 5. S7 attempted to cross-multiply $\frac{3}{4}$ and $\frac{4}{5}$, but mistakenly believed that 20 was derived from 4×4. This reasoning is clearly incorrect, indicating a misunderstanding of multiplication operations with integers. When the researcher provided guidance to help S7 solve the problem, the student appeared even more confused, further highlighting gaps in their understanding of fraction operations and the concept of ordering fractions.

Based on the students' answers and interviews, the causes of their mistakes in solving Question 5 can be summarized as follows: First, students lack good number sense. They are not sensitive to the numerical problems presented, and when faced with mathematical problems involving bounded numbers, they cannot spontaneously generate solutions. According to Griffin (2004), individuals with strong number sense can use their understanding of numbers to solve mathematical problems without being confined by traditional algorithms or procedures. Question 5 is slightly tricky; however, students with strong numerical awareness could quickly and accurately answer it. Second, students lack knowledge of the basic concepts required to solve fraction problems. One alternative solution for Question 5 is to convert the fractions into decimal numbers. The issue, however, lies in the fact that some students do not understand operations with integers.

The obstacles students face in understanding fraction ordering are evident from their answers to Question 5. In this case, students encounter epistemological obstacles, as they fail to identify fractions between $\frac{3}{4}$ and $\frac{4}{5}$ due to errors in performing integer calculations, particularly converting fractions into decimal forms. Furthermore, some students do not understand the concept of determining the Least Common Multiple (LCM) of two numbers. This indicates that students' prerequisite knowledge for solving fraction problems is still weak.

Students' Knowledge of Fraction Operations

The fraction operations referred to in this study include the basic operations of fractions: addition, subtraction, multiplication, and division. The questions provided were not solely procedural; students were also required to explain their answers conceptually using visual representations. The following is an explanation of the findings related to this aspect:

Addition

The understanding of students regarding the operation of addition with fractions can be observed through test question number 6, part (a), on learning obstacles. This question aims to assess how students solve and explain problems related to fraction addition. Figure 7 presents several student responses to question 6(a).

Question 6(a)

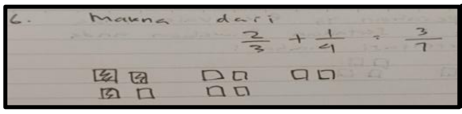
What is the meaning of:

$$\frac{2}{3}+\frac{1}{4}=\cdots$$

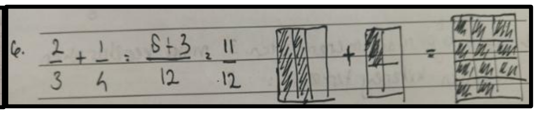
Determine the result and explain how you arrived at the answer using a visual representation! (Lin et al., 2013)

Student Responses in Indonesian

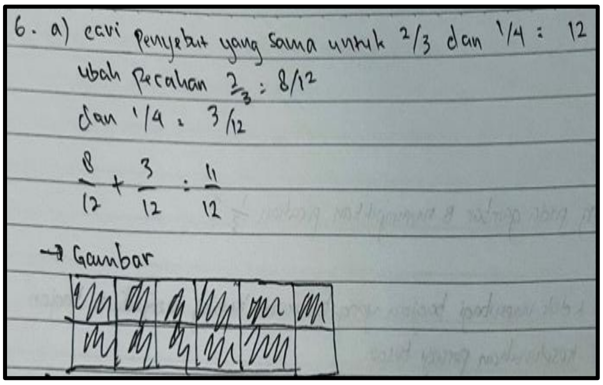
(a)



(c)



(b)



Student Responses in English

(a) The meaning of $\frac{2}{3}+\frac{1}{4}=\frac{3}{7}$

Illustration: See the picture from Figure 7(a) showing the student's response in Indonesian.

(b) Find a common denominator for $\frac{2}{3}$ and $\frac{1}{4}$: 12

Convert the fractions: $\frac{2}{3}=\frac{8}{12}$ and $\frac{1}{4}=\frac{3}{12}$

Add the fractions:

$\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$

Illustration: See the picture from Figure 7(b) showing the student's response in Indonesian.

(c) See the picture from Figure 7(c) showing the student's response in Indonesian.

Figure 7. Student's Answer to Question 6(a)

Based on Figure 1, 11 students answered correctly, 61 students answered correctly but with incomplete explanations, and 1 student provided an incorrect answer. Figures 7(b) and 7(c) show that, procedurally, the operations are correct. However, the students were unable to provide complete explanations as instructed in the question. Most students only provided procedural answers. To gain a deeper understanding of the students' conceptual comprehension, the researcher conducted interviews. The following is an excerpt from the interview:

S2: Because the fractions must have the same denominators first.

P: Why do they need to have the same denominators?

S2: That's just how it is, sir.

For Figure 7(b), S2 correctly solved question 6(a) procedurally but failed to accurately represent the process of adding two fractions using a visual representation. S2 stated that what they did was a fixed rule that must be followed. Therefore, when dealing with fraction addition, that rule must be applied. Similarly, S4, as shown in Figure 7(c), explained that they obtained the answer by finding the least common multiple (LCM) of 3 and 4 but were unable to create a visual representation of the addition process. The following is an excerpt from the interview:

S4: For fraction addition, the denominators must first be the same. So, to make them the same, we look for the LCM. It turns out the LCM of 3 and 4 is 12. Then, 12 divided by 3 equals 4, and 4 times 2 equals 8. And then, we proceed like that. Finally, we get $\frac{11}{12}$. *Since* $\frac{11}{12}$ *cannot be simplified further, we were asked to represent it with a drawing. So, I drew it—* $\frac{3}{2}$ *like this,* $\frac{1}{4}$ *like that, and that's* $\frac{11}{12}$. *That's how it is, sir.*

Next is S1's response, shown in Figure 7(a). This response is quite surprising as it indicates that there are still students who do not know how to solve problems involving the addition of fractions. This means that S1 lacks both procedural and conceptual understanding of fraction addition. To confirm this, the researcher conducted an interview with S1, and the following is an excerpt from the interview:

S1: Actually, as I recall from what you explained before, we should multiply the denominators.
P: Multiply the denominators, right?
S1: Yes.
P: Is the method for adding fractions the same as for adding whole numbers?
S1: I don't think so, sir. It should probably involve cross-multiplication.

Based on the interview excerpt above, S1 was not confident with the response shown in Figure 7(a). During the interview, S1 stated that they recalled the method was to multiply the denominators. However, even after the researcher clarified the question, S1 gave a different answer, suggesting cross-multiplication instead. After the researcher explained the correct method, S1 provided a different response, which turned out to be procedurally correct. The following is an excerpt from the interview after the explanation:

S1: No, the answer for $\frac{2}{3}+\frac{1}{4}$ *is* $\frac{11}{12}$, *while* $\frac{2}{3}-\frac{1}{4}$ *is* $\frac{5}{12}$ *now.*
P: $\frac{11}{12}$, *right? And then this becomes* $\frac{5}{12}$, *correct?*
S1: Yes.
P: Why did your answer change? What made you think, "Oh, this is the answer"? Why?
S1: Because it was explained earlier that you actually have to use cross-multiplication, not just directly adding or subtracting when dealing with fractions.

Based on the students' responses and interview results, the causes of student errors in answering question 6(a) can be summarized as follows: First, lack of conceptual understanding: Students do not have a strong conceptual grasp, particularly regarding representing addition operations in a visual form. This is evident even when the students demonstrated correct procedural understanding. Second, Incorrect procedural understanding: Students' errors in solving addition operations with fractions stem from flawed procedural knowledge, making it unlikely for them to develop good conceptual understanding. Given that the topic of visual representation in fraction operations is considered a challenging aspect of fractions, this aligns with Ball's (1990) explanation. Ball stated that although students may be able to solve fraction operation problems mathematically, very few are capable of providing visual explanations.

Subtraction

Students' understanding of fraction subtraction can be observed in test question 6(b), which is part of the learning obstacle assessment. This question aims to evaluate how well students can solve and explain problems related to fraction subtraction. Generally, the issues encountered by students in question 6(b) are similar to those in question 6(a), as the methods for solving both problems are essentially the same. Figure 8 illustrates several student responses to question 6(b).

Question **6(b)**

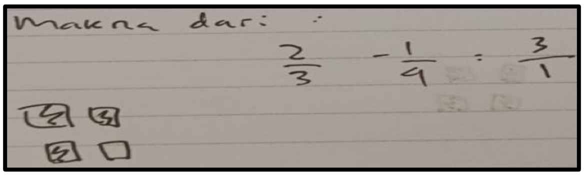
What is the meaning of:

$$\frac{2}{3} - \frac{1}{4} = \cdots$$

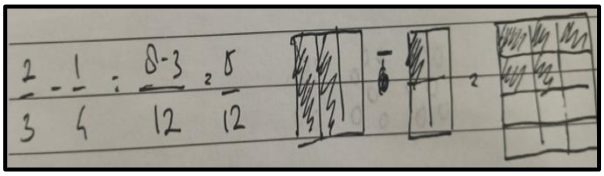
Determine the result and explain how you arrived at the answer using a visual representation! (Lin et al., 2013)

Student Responses in Indonesian

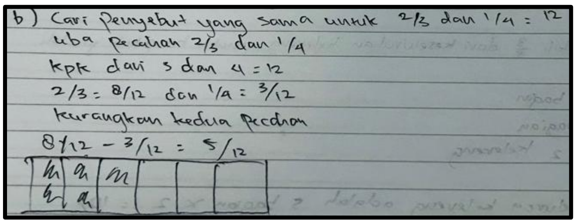
(a)



(c)



(b)



Student Responses in English

(a) The meaning of $\frac{2}{3} - \frac{1}{4} = \frac{3}{1}$

Illustration: See the picture from Figure 8(a) showing the student's response in Indonesian.

(b) Find a common denominator for $\frac{2}{3}$ and $\frac{1}{4}$: 12

Convert the fractions: $\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$

Subtract the fractions:

$$\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$$

Illustration: See the picture from Figure 8(a) showing the student's response in Indonesian.

(c) See the picture from Figure 8(c) showing the student's response in Indonesian.

Figure 8. Student's Answer to Question 6(b)

Based on Figure 1, 11 students answered correctly, 61 students answered correctly but with incomplete explanations, and 1 student answered incorrectly. Figure 8(b) and (c) demonstrates that the procedural operations were correct, but the students were unable to provide complete explanations as required by the question. In question 6b, students were also asked to represent the subtraction operation in a pictorial form. From the interview results, the explanations provided by the students were almost identical to how they solved question 6a. Similar to S1's response in question 6(a), S1 also provided a surprising answer in Figure 8(a). However, after confirmation, the interview transcript matched the response to question 6(a), where the student provided a procedurally correct answer after receiving clarification from the researcher.

Based on the students' responses and interviews, it can be concluded that the errors made by students in answering question 6(b) are almost identical to those in question 6(a). These errors suggest that students lack a strong conceptual understanding, particularly regarding the representation of subtraction operations in pictorial form, even though their procedural understanding was correct.

Question 6 reveals the presence of an epistemological obstacle due to the students' limited understanding of the meaning of operations involving fractions. While pictorial interpretations of fraction operations had been taught previously, this did not enable students to fully grasp the meaning of these operations. Consequently, students' knowledge was confined to the procedural mechanisms for solving addition and subtraction operations.

Multiplication

Students' understanding of fraction multiplication can be observed through test question 7(a) on learning obstacles. This question aims to assess how students perform and explain problems related to fraction multiplication. Figure 9 illustrates several student responses to question 7(a).

Question **7(a)**

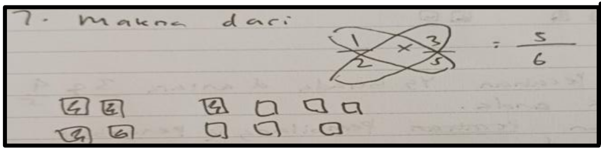
What is the meaning of:

$$\frac{1}{2}x\frac{3}{5} = \cdots$$

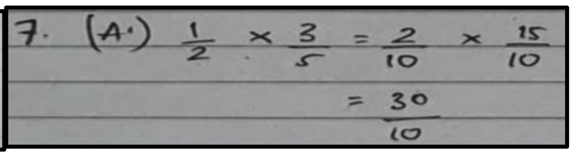
Determine the result and explain how you obtained your answer using a pictorial representation! (Lin et al., 2013)

Student Responses in Indonesian

(a)

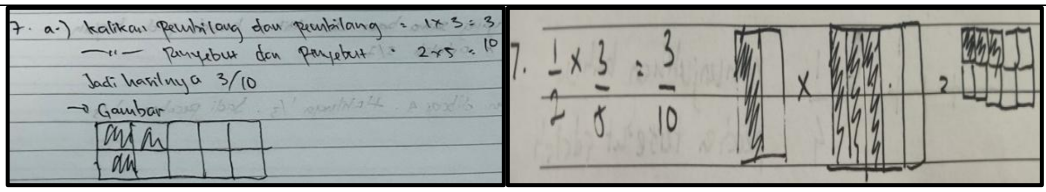


(c)



(b)

(d)



Student Responses in English

(a) The meaning of:

$$\frac{1}{2} x \frac{3}{5} = \frac{5}{6}$$

Illustration: See the picture from Figure 9(a) showing the student's response in Indonesian.

(b) Multiply the numerator with the numerator: 1 × 3 = 3

Multiply the denominator with the denominator: 2 × 5 = 10

Therefore, the result is $\frac{3}{10}$.

Illustration: See the picture from Figure 9(a) showing the student's response in Indonesian.

(c) See the picture from Figure 9(c) showing the student's response in Indonesian.

(d) See the picture from Figure 9(d) showing the student's response in Indonesian.

Figure 9. Student's Answer to Question 7(a)

Based on Figure 1, 65 students provided correct answers with incomplete explanations, 3 students answered incorrectly, and 5 students did not respond. Figure 9 (b and d) demonstrates that while the procedural operations were correct, the students were unable to provide complete explanations as required by the instructions. They could not explain multiplication operations in pictorial form. Most students were only able to provide answers like those shown in Figure 9 (b) and (d). To better understand students' conceptual understanding, the researcher conducted interviews and obtained the following excerpts:

S2: For fractions, like we said earlier, we just multiply directly, meaning like, numerator by numerator and denominator by denominator.

S4: For multiplication, you just multiply directly, sir, not like addition or subtraction. So, you just do 1×3=3, 2×5=10, and then I draw it again, like my understanding, like for $\frac{1}{2}$, oh, this is the picture, and for $\frac{3}{5}$, oh, this is the picture, and that's it.

For Figure 9(b), S2 was only able to answer question 7(a) procedurally. The explanation provided by S2 during the interview was the same; they did not know how to create a pictorial representation for multiplication operations. The image created by S2 on their answer sheet was merely an illustration of the fraction $\frac{3}{10}$. A similar explanation was given by S4, whose answer is shown in Figure 9(d). S4 admitted to representing each fraction individually, rather than the operation of fraction multiplication. This can be seen from the interview excerpt of S4 above.

Meanwhile, surprising answers were given by S1 and S3. S1 solved the problem using cross-multiplication, and S3 attempted to equalize the denominators, as is done in addition and subtraction operations with fractions. To delve deeper into the reasoning behind S1's answer, the researcher conducted an interview, and the following is an excerpt from the interview:

S1: It's the same as the method, sir, where for fractions you cross-multiply, but it shouldn't be like that.

P: How should it be, then?

S1: It should be that you just multiply directly, not cross-multiply like that.

P: I see, so just multiply directly. What would that look like? So, what is the answer for question 7 then?

S1: For $\frac{1}{2}\times\frac{3}{5}$*, I shouldn't cross-multiply. I should just multiply directly, 1×3=3, 2×5=10, so the answer is* $\frac{3}{10}$*.*

Based on S1's response excerpt above, S1 confirmed that the answer written in the answer sheet was incorrect and then corrected it to the proper procedural solution. A different scenario emerged with S3 during the interview. S3 mentioned using cross-multiplication, but this differed from what was written on the answer sheet. The researcher concluded that S3 faced issues understanding the concept of fraction multiplication, as further questions from the researcher led to increasingly confusing answers. Evidence of this confusion can be seen in the interview excerpt, where S3 struggled to differentiate between denominators and numerators:

S3: Because we were told to multiply, so I multiplied the...

P: Yes, and then multiplied what?

S3: I multiplied the numerator. Oh, here we were told to cross-multiply for the numerator, so we multiply the top by the bottom, meaning we multiply $\frac{1}{2}$ *and 3 x 5 the same way.*

P: So, is there any difference between this multiplication and addition? Or is the method the same? Is the method for addition and multiplication of fractions the same?

S3: It's different.

P: Where is it different?

S3: For this one, I multiplied 3×3, uh, cross-multiplied.

P: But why is the denominator here 10?

S3: Because I multiplied 2 × 5.

P: Okay, so the difference lies here, right? For addition, earlier you cross-multiplied the top. What is the top called? The denominator, right?

S3: Yes, I don't know.

Based on the results of students' answers and the interviews conducted, it can be concluded that in the context of fraction multiplication, not all students were able to solve the given problems procedurally, and many still lacked the ability to conceptually explain the multiplication process using graphical representations. Furthermore, none of the respondents demonstrated an understanding of the meaning or significance of fraction multiplication. This lack of understanding stems from the absence of prior instruction regarding the conceptual meaning of multiplying fractions.

Understanding the meaning of fraction multiplication is inherently complex and should be a priority for educators to explain in a clear and in-depth manner (Webel & DeLeeuw, 2016). Moreover, this difficulty is a common challenge faced by learners in general. This finding emphasizes the need for improved pedagogical strategies that not only focus on procedural correctness but also foster a deeper conceptual understanding of mathematical operations like fraction multiplication.

Division

The understanding of students regarding the operation of division with fractions (*fraction division*) can be analyzed through the *learning obstacle* test question number 7, part b. This question aims to assess how students solve and explain problems related to the operation of division with fractions. Figure 10 illustrates several student responses to question 7(b).

Question **7(b)**

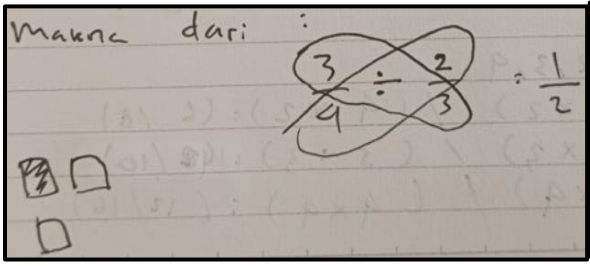
What is the meaning of:

$$\frac{3}{4} \div \frac{2}{3} = \cdots$$

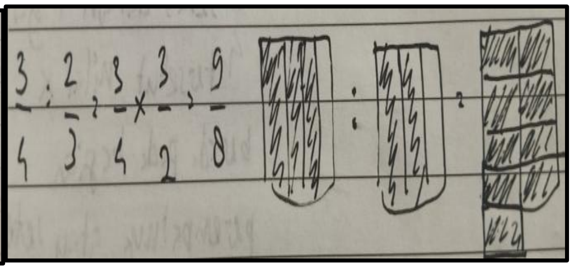
Determine the result and explain how you derived the answer using a pictorial representation! (Lin et al., 2013)

Student Responses in Indonesian

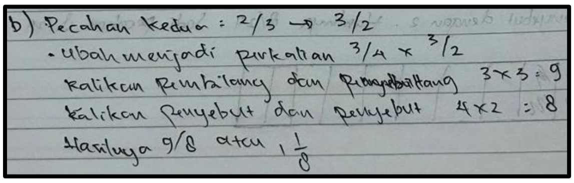
(a)



(c)



(b)



Student Responses in English

(a) The meaning of:

$$\frac{3}{4} \div \frac{2}{3} = \frac{1}{2}$$

Illustration: See the picture from Figure 10(a) showing the student's response in Indonesian.

(b) Second Fraction: $\frac{2}{3} \rightarrow \frac{3}{2}$

Change into multiplication: $\frac{3}{4} \times \frac{3}{2}$

Multiply the numerators: $3 \times 3 = 9$

Multiply the denominators: $4 \times 2 = 8$

The result is $\frac{9}{8}$ or $1\frac{1}{8}$.

(c) See the picture from Figure 10(c) showing the student's response in Indonesian.

Figure 10. Student's Answer to Question 7(b)

Based on Figure 1, there are 65 students who answered correctly but provided incomplete explanations, 3 students who answered incorrectly, and 5 students who did not respond. Figures 10 (b) and (c) show that while the procedural operations were correct, the explanations provided in each answer were incomplete as per the instructions. To better understand the students' conceptual comprehension, the researcher conducted interviews. Excerpts from the interviews are as follows:

S2: Why is it like that? Why multiply here, friends? I think at that time, I was already confused. Why was it multiplied there? Oh no, I was half-conscious when I worked on this, sir.

S4: Oh, for division, what do we do next? First, we multiply. I mean, we change it into multiplication, and then in the last step, for example, this $\frac{2}{3}$ *is flipped into* $\frac{3}{2}$*. That's how we get 3 × 3 = 9 and 4 × 2 = 8, like that, sir.*

For Figure 10(b), S2 struggled to explain why they answered as they did on the answer sheet. Although their answer was procedurally correct, S2 did not understand the concept and was unable to provide an explanation of their written response. This lack of understanding also resulted in S2 being unable to create a pictorial representation. Meanwhile, S4 was only able to solve Question 7b procedurally. The explanation provided during the interview was similar to their written response, which included a representation of the fractions. However, S4 could only represent each fraction individually and failed to depict the division operation between the fractions. In contrast, S1's answer differed from those of S2 and S4, as shown in Figure 10(a). Below is an excerpt from S1's interview:

S1: Hmmm, maybe division. Yes, it's multiplied crosswise.

P: Multiplied crosswise like this, so it's correct, right? If it's multiplied crosswise, does it mean two times four when multiplied crosswise?

S1: Yes.

P: Or is it divided by two directly?

S1: Uh, sir, I first multiply three, then divide by three, which gives one. Four divided by two equals two, so it's one-half.

Based on the interview excerpt above, S1 demonstrates a different understanding of fraction division. Initially, S1 explained that the division of fractions is performed by "cross multiplication," but in the subsequent explanation, S1 actually performed "cross division." This highlights that S1 has not yet fully grasped the concept of fraction division automatically. This is further supported when the researcher introduced a new problem related to fraction division to test S1's explanation. Below is the interview excerpt:

P: Let's try this: draw the solution. But, for example, if I give you a new problem, let's say $\frac{1}{2} \div \frac{1}{3}$*, what's the result?*

S1: (thinking) $\frac{1}{2} \div \frac{1}{3}$, $\frac{1}{5}$*, sir, or* $\frac{3}{2}$.

P: $\frac{1}{5}$ *or* $\frac{3}{2}$*?*

S1: Yes.

P: So, there are two answers?

S1: I'm confused whether to choose $\frac{1}{5}$ *or* $\frac{3}{2}$.

P: Okay, why did you get $\frac{1}{5}$*? Where does it come from?*

S1: For $\frac{1}{5}$*, I just multiply directly.* $\frac{1}{2} \times \frac{1}{3}$*, so numerator with numerator, denominator with denominator. That's 1×1=1, and 2 x 3 = uh, 6 actually 2 x 3 = 6.*

P: So, it's 6, not 5?

S1: Uh, yes, 2 x 3 = 6. Yes.

P: So, here it's 6. Okay, and where does the $\frac{3}{2}$ *come from?*

S1: I tried flipping it, sir.

P: Flipping it, meaning?

S1: . $\frac{1}{2} \times \frac{3}{1}$.

P: Okay, and the result?

S1: $\frac{3}{2}$.

Based on the students' answers and interview results, it can be concluded that students face numerous challenges when solving problems related to fraction division operations. First, some students are unable to perform fraction division procedurally. Second, many students struggle to represent fraction division operations in visual form.

The obstacles students encounter in understanding multiplication and division operations with fractions are evident from their answers to question 7, parts (a) and (b). The issues observed in the cases of fraction multiplication and division are similar to those found in fraction addition and subtraction. While students can correctly perform procedural operations, they are unable to represent these operations visually. In question 7, epistemological obstacles arise due to the limited understanding of how to solve multiplication and division operations in visual form. Furthermore, in some cases—particularly in division—students only recognize that when faced with a fraction division problem, they must convert it into a multiplication problem. However, they lack understanding of the correct procedure for inverting the divisor.

Another epistemological obstacle arises because none of the students explained the unique characteristics of fraction division. It is important to note that the concept of division in fractions is very different from the concept of division in whole numbers, which aligns with real-life contexts. In everyday situations, dividing one cake among two people means each person receives half a cake, with the result always being smaller than the initial quantity. However, in fraction division, the result can sometimes be greater than the fraction being divided.

Implications for Elementary Mathematics Education

The findings of this study highlight the importance of identifying epistemological obstacles in understanding the concept of fractions, particularly for prospective teachers in Primary School Teacher Education programs. A deep understanding of these obstacles can directly contribute to the development of more effective pedagogies, both in classroom instruction and curriculum design. (1) Strengthening concept-based curriculum: The discovery of these learning obstacles recommends that mathematics curricula, especially in the topic of fractions, should be designed to introduce fraction concepts more comprehensively. Utilizing visual aids, such as number lines or concrete models, can help students build a more intuitive and flexible understanding. (2) Enhancing teachers' Professional competence: Teachers need to be equipped with training that not only focuses on delivering content but also on identifying and anticipating students' epistemological obstacles. Such training can improve teachers' capacity to adapt instructional strategies to meet students' needs. (3) Promoting discovery-based learning approaches: Providing opportunities for students to explore and discover mathematical concepts independently can encourage more meaningful learning. This approach can also enhance students' critical thinking and problem-solving skills.

Conclusion

Based on the explanations provided earlier, the following conclusions can be drawn regarding students' knowledge of fractions, which may hinder their understanding of the topic:

1. Limited understanding of fraction concepts. Students have a very restricted understanding of fractions, including the concept of fractions as part of a whole (part-whole), as part of a set (part-set), as a unit, and as a number on a number line. While many students are familiar with or have encountered these concepts, they do not fully understand them and struggle to implement them in the given problems.
2. Lack of diverse fraction problems. Students often struggle with solving non-routine fraction problems due to the limited variety of fraction exercises they are exposed to. Most students can only answer routine questions related to fractions.
3. Difficulty integrating prior knowledge. Students find it challenging to connect what they have learned about fractions with their prior knowledge of prerequisite mathematical concepts.
4. Struggles with specific fraction scenarios. Students face difficulties understanding certain aspects of fractions, such as equivalence and representing fractions in pictorial form.
5. Lack of sensitivity to mathematical problems. Students are less responsive to numerical problems, and when faced with time-constrained mathematical tasks, they struggle to generate solutions spontaneously.
6. Focus on procedural learning. Students tend to prioritize procedural learning in mathematics, often failing to deeply understand the meaning behind fraction operations. This makes it difficult for them to develop conceptual knowledge, especially when dealing with more complex situations. Consequently, students struggle to solve mathematical problems that deviate from routine examples, even when the underlying concepts remain the same.
7. Decreased motivation. A habitual reliance on solving relatively easy, routine problems results in reduced motivation when faced with challenging tasks. Students tend to give up easily when they forget previously taught concepts and fail to explore their reasoning and experiences in such situations.
8. Procedural proficiency without conceptual understanding. While many students are capable of solving fraction problems procedurally, they often fail to explain the operations conceptually or represent them using pictorial forms.

Based on the research findings, the identified learning obstacles, particularly the epistemological obstacles experienced by students when answering questions related to fractions, are as follows:

1. Limited understanding of fraction concepts. Students have a very restricted comprehension of fractions, as they do not fully understand the concept of fractions as part of a whole (part-whole), as part of a set (part-set), as a single unit (unity), and as a number on a number line.
2. Inadequate understanding of prerequisite topics. Students struggle with prerequisite concepts such as the Least Common Multiple (LCM) and division of whole numbers.

3. Lack of understanding of fraction properties. Students do not have a solid grasp of the ordering properties of fractions, equivalent fractions, and fraction operations.
4. Inability to represent fractions pictorially. Students are unable to perform fraction operations using pictorial representations.

Recommendations

While this study provides significant insights, several limitations are acknowledged, along with suggestions for future research: (1) Contextual limitations: This study was conducted with Primary School Teacher Education programs students at a single location. To achieve more generalizable results, similar research could be conducted in various institutions and involve students from more diverse backgrounds. (2) Focus on fraction topics: This study focused on epistemological obstacles in understanding fractions. Future studies could extend this analysis to other mathematical topics, such as geometry, proportions, or algebra, to examine whether similar obstacles emerge in these concepts. (3) More comprehensive methodological approaches: This study primarily used qualitative methods. Future research could involve a combination of qualitative and quantitative methods, such as controlled experiments or pre-and post-test data analysis, to measure the effectiveness of instructional interventions. (4) Longitudinal studies: It is recommended that long-term studies be conducted to evaluate the impact of instructional strategies based on identifying epistemological obstacles on students' mathematical understanding. These longitudinal studies would provide deeper insights into the long-term effectiveness of teaching strategies.

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