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New Studies in Mathematics Trails

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Abstract

The paper is about new technological approaches to mathematics trails and current developments in the empirical research on mathematics trails. For a long time, maths trails have been used for the popularization of mathematics. Therefore previous research has focused on motivational aspects and the change of mathematical beliefs. With the successful shift from pencil-and-paper maths trails to mobile devices and new technology came new ways to collect data and perform research on maths trails. Besides the classical maths trail activity, students show more motivation towards mathematics because of mobile devices and tasks that illustrate the usefulness and possible mathematics applications. Furthermore, walking a maths trail leads to a better mathematical performance than having a regular lecture without going outdoors or such activity. It affects long-term memory and intrinsic motivation positively.

Keywords: Handheld devices, Mathematics activity, Mathematics trail, Outdoor education

Introduction

At first sight, a mathematics trail seems to be a very non-digital experience for creators and learners. As described in the 1980s, the classic maths trail is activities outside the classroom, where a small group walked through the neighborhood and solved mathematical problems at specific objects (Blane, 1989; Lumb, 1980). On second thought, mathematics trails will prove to be an excellent example of how a non-digital activity can be augmented with digital technology and become a more motivating and better learning environment. This paper will show the first steps towards a symbiosis of analogue and digital learning and how they can complete and benefit from each other. Two studies have been conducted to examine these benefits, the first one in Indonesia and the second one in Germany.

Why running a maths trail? It is recommended, for example, by the British department of education and skills to do more lessons outside (DfES, 2006). Maths trails are an excellent opportunity to integrate an outdoor lesson into the mathematics curriculum. Walking outside exposed to daylight may help to prevent the effects of prolonged sitting (Lurati, 2018) and myopia (short-sightedness), as suggested by the World Health Organisation (2015). Mathematics trails are a great way to popularize mathematics (Blane, 1989). So, it is about health and fun. Nevertheless, what is it about the learning outcome? Is it worth the effort to create a maths trail and go outside with the pupils?

Digital technology can reduce the effort to create a maths trail. The significant change from the "old" web technologies to Web 2.0 was social media possibilities. The internet became a participatory web, where users generate content and communicate (Blank & Reisdorf, 2012). The participatory aspect is especially interesting for educational resources since key users like teachers getting paid from public money and is not dependent on making money with their content. On the contrary, most teachers are willing to share their material free with others. Geogebra Tube, for example, has millions of free worksheets generated from users. Authoring of maths trails with digital tools is not the focus of this article; nevertheless, the following articles touching this theme: Gurjanow, Ludwig & Zender (2017, 2018) and Jablonski, Ludwig & Zender (2018).

Since a maths trail is a part of outdoor education, it makes sense to look at mobile devices, like Wijers, Jonker and Drijvers (2010) are recommended to use to support and enrich outdoor learning. Going on a maths trail could greatly benefit from using mobile devices because they allow learning to occur in an authentic context and extend to real environments. Mobile devices can be taken to the objects and support the users with maps, hints, feedback and communication tools. Although mobile devices and computers are widely used in every aspect of our daily lives (especially among pupils), they played a minor role in education (Chen & Kinshuk, 2005). However, the distribution of mobile devices and mobile internet became unique. Today 5.2 billion people have a mobile device, of which 3.8 billion have access to mobile internet. The GSM Association expects 5 billion people to have a mobile internet connection in 2025 (GSM Association, 2020).

MathCityMap

At the Goethe University of Frankfurt am Main we started the MathCityMap project (MCM), which combines traditional maths trails with the opportunities of new technologies as listed above. It was not the first attempt to connect new technology with maths trails. In the year 2000, the US Department of Education and Texas Instruments published a website called "The National Math Trail", where users could upload their maths trail guides as .doc or .pdf files (original site was: http://www.nationalmathtrail.org/). Something similar happened sometime later in Canada with "Canadian Math Trail" (https://brocku.ca/cmt/). Both websites have been active from 2000 to 2002 but then stopped generating new content, "The National Math Trail" vanished in 2010 and could only be found via the Internet Archive.

Law and So (2010) used mobile devices to read QR Codes with tasks placed at the object. Chen (2013) used the chat functions of Google Buzz to allow pupils to communicate with their teacher during a maths trail activity.
In addition to these approaches, the MathCityMap project (MCM) was established at the Goethe University of Frankfurt in 2012 (Jesberg & Ludwig, 2012). Still, it took until 2016 to finally launch a web portal and a mobile application. MathCityMap now provides users with a web portal as a GUI for a database of maths trail tasks and routes and an application for smartphones (iOS and Android) to compile this data into a mobile trail guide (see figure 1). The app gives feedback on solutions (wrong/right), and the users can display hints. Following Aebli's (1983) ideas, giving direct feedback is essential, so no wrong solution is without comment.

Furthermore, with a connection between the web portal and the app, it is now possible to create a digital classroom, a web session where the pupils can join their smartphones (Ludwig, Baumann-Wehner, Gurjanow, Jablonski, 2019). Simultaneously, the teacher can communicate via the web portal and see their progress and location via GPS (if shared by the pupils). MathCityMap has been part of the Erasmus+ projects MoMaTrE and MASCE[3].

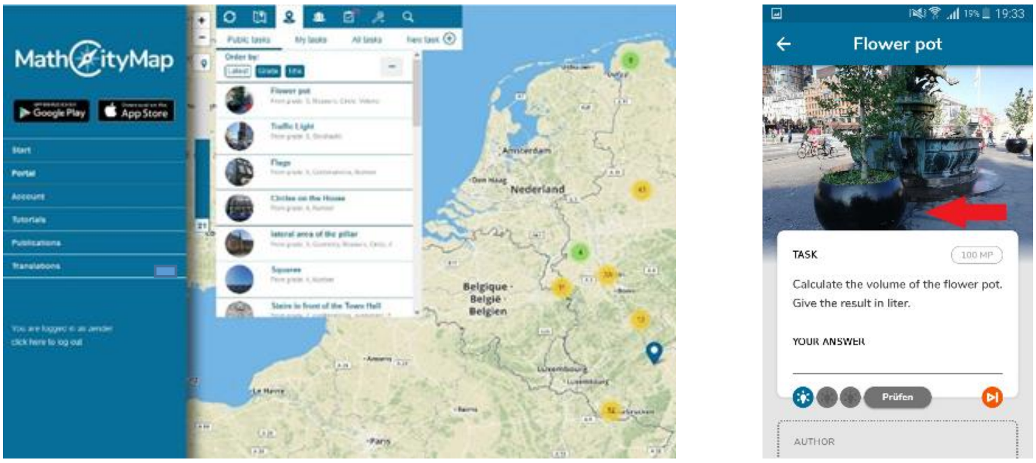

Figure 1. MathCityMap Web Portal (Left) and Smartphone Application (Right)

Theoretical Framework

The findings in this paper are mainly related to the learning of mathematics using the MCM app. Therefore, the aspect of teaching mathematics using MCM will not be at the core of the following paragraphs. We recommend the article by Barbossa and Vale (2020).

Mobile Learning of Mathematics

Park (2011) defines mobile learning as learning while on the move by using portable electronic devices. The learning is facilitated by easy-transportable digital tools, such as smartphones or tablets. In contrast to learning inside the classroom, pupils are not bound to a single fixed location. If, in addition to using a digital tool, the learning is also associated with an object of the real world, then Lonsdale, Baber, Sharples and Arvanitis (2004) speak of context-aware ubiquitous learning (u-learning). U-learning is considered a subcategory of mobile learning. Since MCM uses mobile devices to facilitate mathematics learning at predefined objects of the real world, it can be categorised as a tool for u-learning. Using real-life objects for mathematics learning holds many theoretical benefits regarding students' motivation and performance, which will be elaborated more deeply in the following paragraphs.

The role of technology in the outdoor context helps pupils become mathematically active by presenting tasks prepared by the author of a maths trail. Therefore, this approach is also called *artefact-led* mobile learning (Donevska-Todorova, 2020). Beyond that, the MCM app supports learners in their independent solution process by offering predefined hints and validating entered answers based on a stored solution set. Since the studies presented in this article generally focus on pupils' learning outcomes and motivation using a digital tool for the outdoor learning of mathematics, the support and feedback system's contribution remains examined.

Motivation

The self-determination theory by Ryan and Deci (2000) makes an essential distinction between two types of motivation: intrinsic and extrinsic motivation. According to Fredricks, Blumenfeld and Paris (2004), intrinsic motivation holds desirable learning potentials, reflected in personal, cognitive and emotional engagement. Activities need to fulfil three psychological needs to be perceived as intrinsically motivating: autonomy, competency and relatedness (Ryan & Deci, 2000). Using technology to learn mathematics outdoors also means giving students more control over the learning process. Compared to regular maths classes, they have to make more decisions, such as reaching the task location, the approach to solve the task and how to interact with the MCM application, which increases autonomy. The MCM app supports pupils with feedback and stepped hints to help them work independently on the maths trail tasks and feel more competent. However, this support does not automatically mean that every pupil will be able to solve all tasks. The social form of group work meets the psychological need for relatedness and is often the first choice for out-of-school learning. It can be concluded from the above that u-learning with MCM offers a good starting point for motivating learning.

Learning Growth

Besides the motivational advantages of u-learning, other potentials speak for an increase in learning when completing a maths trail. First, objects of the real world can be experienced first-hand with many senses. In contrast to secondary experiences, i.e. second-hand experiences conveyed utilizing media or narratives, primary experiences are more memorable because they are embedded in an associative environment (Kovalik & Olsen, 1994). Especially the connection between enactive actions (measuring objects), iconic representations (e.g. creating a sketch of the object) and the symbolic representation (e.g. formula of the volume of an object) is considered valuable for learning mathematics outdoors (Ludwig, Jesberg & Weiß, 2013). Second, at out-of-school places of learning, pupils usually find a complex initial situation, which offers them freedom for their discoveries and active exploration of the subject matter (Killermann, Hiering & Starosta, 2008). Discovering and exploring are typical activities of constructivist learning environments, which are said to have the potential to produce knowledge that can be accessed for longer (Karpa, Lübbecke & Adam, 2015). Third, it is easy to pose authentic questions using real-life objects. Using the MCM application, additional information about the object can further enhance the task's authenticity. Vos (2015) claimed that an authentic task in mathematics should have an out-of-school origin and be certified by experts as a task of their field. The real-life object on a maths trail gives the out-of-school origin. Our case's certification is understood to ask the obvious question towards the object, not an absurd one. For an advertisement pillar (a cylindrical object), it is obvious to ask for the lateral area. That is the area the advertisement is displayed. However, it would sound absurd to ask for the surface area, which got nothing to do with its function. However, the surface area can be calculated with the same measurements as the lateral area.

Previous Research on Mathematics Trails

Over the last 30 years, there have been a few studies on mathematics trails. Probably the first was done by Kaur (1992) in Singapore. Kaur (1992) reported that pupils are more motivated by maths trails since they found it more meaningful and fun than regular classroom lessons. Later, Toh and Lim (2006) let pupils create maths trails in Singapore. These pupils have had fun and gained a new perspective on mathematics. Callenberg and Johansson Andersson (2014) did interviews with pupils after running a maths trail in Sweden. The pupils stated that a maths trail is fun and helped them to discover mathematics in the environment.

Finnland, Rikala and Kankaanranta (2014) are the first to research a technology-supported maths trail. They were inspired by Law and So (2010) and placed QR Codes on objects which led to mathematical tasks. The pupils' teacher reported that the pupils did better in the next exam although they had a shorter time to practise than usual. In Germany, Buchholtz (2017) and Buchholtz and Armbrust (2018) did two studies with maths trails created with the mobile app Actionbound. In both studies, the pupils have been highly motivated through the app and maths trail.

Since maths trails have been rooted in the popularization of mathematics, the previous studies had focused on aspects of motivation and beliefs. All of them could find positive effects amongst all age groups from primary to middle and secondary school. However, all of them were qualitative, with a relatively small number of participants (20 to 50). Since motivation and performance are related (Chiu & Xihua, 2008), it seems obvious to research the effects of maths trails on pupils' performance (More details on the research and historical development on maths trails can be found at Zender (2020).

There is a gap. What is missing is quantitative research, especially on the performance regarding mathematics trails. We will now present the studies of Cahyono (Indonesia) and Gurjanow and Zender (Germany) concerning pupils' performance and motivation.

Research Question

How does a mobile app supported maths trail affect the learning outcome and motivation of pupils?

Method

The mathematics education group of the Goethe University of Frankfurt A has conducted two studies. The first one took place in Semarang, Indonesia (Cahyono, 2017). The second one took place in Frankfurt am Main, Germany (Zender, 2019). The concept of ecological validity primed both studies. The studies should take place as close to the actual situation in which maths trails are used as possible so that the results reflect the potential effects of maths trails in schools. Consequently, the Indonesian study strongly involved the teachers in creating maths trail tasks and the German study was done during the school lessons inside the curriculum and not as an add-on.

The Indonesian study involved 520 pupils from seventh to ninth grade and nine of their teachers from nine different schools. The research was conducted from 2014 to 2016. Cahyono programmed an own version of the MathCityMap app, with similar functions and behavior, since the MCM project has not launched the official version to that point (see Figure 2).

Figure 2. Indonesian Version of the App for the Indonesian Study

The pupils were divided into an experimental group of 272 pupils and a control group of 248 pupils. We choose one class for both of the groups from each school. All pupils wrote a pretest and a post-test. The same teacher taught experimental and control groups at each school with the same topic and subject matter but with different interventions. The experimental group took part in a maths trail for two or three lessons (45 minutes each), while the control group had regular lessons. Before and after the maths trail, the pupils completed a questionnaire with the Situational Motivation Scale (SIMS), which based on the self-determination theory (Guay, Vallerand, & Blanchard, 2000) based on the works of Ryan and Deci (2000). Every school had its trail, created by the researcher in cooperation with teachers from that school. The pupils ran the maths trail in groups of three. Four schools ran the maths trail a year later, and the pupils completed the SIMS questionnaire again.

The German study involved 629 pupils from grade nine and 23 of their teachers. The research was conducted in 2016 and 2017 and included topic-specific trails about cylindrical tasks. We choose a control/experimental group design. Therefore, the pupils had to be divided into these groups. The division was done after a general test on mathematics knowledge to create two equally good groups. We choose the VERA8 test from 2010 for that purpose. VERA8 is a nation-wide exam in eighth grade to test the mathematics knowledge of all pupils in Germany. It takes 90 minutes and consists of various tasks regarding competencies, themes and levels of difficulty.

The control group stayed in the classroom and had regular lessons about cylinders, including working with the textbook (see figure 3). The experimental group had four lessons less (180 minutes) than the control group. During this time, they went outside twice for a maths trail at the university for 90 minutes (for example, see figure 4). The maths trail included tasks corresponding to typical textbook task on cylinders (asking for the volume, surface area and lateral area of a cylinder). After the maths trail, they completed a survey from the Intrinsic Motivation Inventory (IMI) with the subscales enjoyment, usefulness, perceived competence, and pressure connected to Ryan and Deci's motivation theory (2000).

The experimental group ran the maths trail with the MathCityMap application on smartphones owned by the university. All actions and inputs have been logged and used for research with the knowledge and consent of the participants (figure 5 shows such a log file). The log file consists of information like where have the pupils been (geodata), at which time (timestamp) and what have they entered in the MCM app. It was logged when a task or a hint was opened, when a solution was entered, which solution was entered and if that solution was wrong or right. From this data, it is possible to recreate the sequence of events along the maths trail. Learning Analytics became possible with this data.

After the lessons on cylinders, both groups, the control and experimental group, wrote the same test with cylinder tasks which have been various textbook tasks. Half a year later, the pupils wrote a follow-up test with the same tasks as the first test.

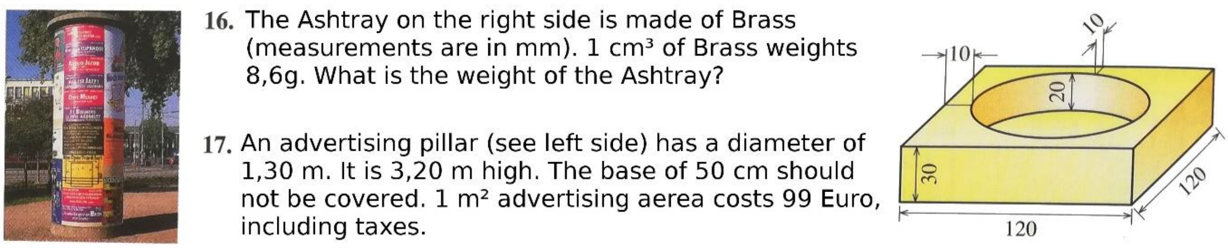

Figure 3. Part of a German Textbook Page with Tasks on Cylinders (Translated)

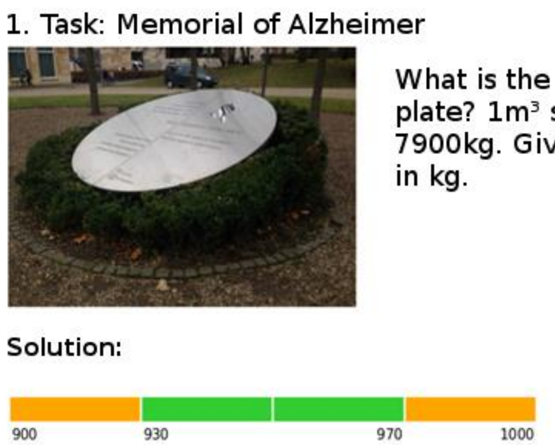

Figure 4. Sample Tasks from the Study about the Weight of a Cylindric Plate

39	2017.06.21.10.29.48	Starting Map View
40	2017.06.21.10.29.57	Start from task: (814) Steinkreis
41	2017.06.21.10.29.59	Open preview dialog for task: (814) Steinkreis
42	2017.06.21.10.30.02	Close preview dialog for task: (814) Steinkreis
43	2017.06.21.10.32.57	Open preview dialog for task: (814) Steinkreis
44	2017.06.21.10.33.03	Open task view: (814) Steinkreis
45	2017.06.21.10.33.04	Task Details open: (814) Steinkreis
46	2017.06.21.10.41.33	Show hint1 for task: (814) Steinkreis
47	2017.06.21.10.42.11	Show hint2 for task: (814) Steinkreis
48	2017.06.21.10.43.02	Show hint3 for task: (814) Steinkreis
49	2017.06.21.10.51.35	Checking answer '1.61' for task: (814) Steinkreis
50	2017.06.21.10.51.35	Task solved' with 100 points for task: (814) Steinkreis

Figure 5. Sample from a Log File. Every Action Performed by the App is Listed with a Timestamp.

Results

The Indonesian study found no significant difference between the control and experimental group regarding the pretest on mathematics ($p = .35$ for a two-sided t-test). However, the post-test has a significant difference between the two groups ($p < .000$, $d = 1.2$). However, it is worth changing the point of view and do a paired t-test for both groups between the pretest and post-test for the control group ($p < .000$, $d = 0.354$) and the experimental group ($p < .000$, $d = 1.542$) (see Table 1).

Table 1. Differences between the Pretest and the Post-test (Indonesian Study)

Group		*N*	*M*	*SD*	*P*	*D*
Control	Pre	248	62.04	11.94	0.000	0.354
	Post	248	66.29	12.06		
Experimental	Pre	272	63.01	11.73	0.000	1.542
	Post	272	79.28	9.22		

After the performance, the motivation scores are also impressive. The Self-determination Index at the beginning had a mean value of -2.568 and a standard deviation of 0.183. Overall, 218 of all pupils had a negative score. After the maths trail activity, the mean SDI score became 7.318, with a standard deviation of 2.926 and not even one person with a negative score. The four schools that made the maths trail again after one year got nearly the same results as before. A two-sided t-test revealed a significant difference between before the maths trail and after, but the second trail did not raise the SDI again; it remains on a high level. In an open-answer questionnaire, nearly a quarter of the pupils stated that mobile technology was the most motivating factor. They reported how much fun they had to locate the tasks, and so on.

The German study could first show that the division into the control and experimental group succeeded in general mathematical performance. The scores of the VERA8 test are normally distributed, and a two-sided t-test could not show a significant difference between the two groups ($p = .93$). Comparing the control and experimental group in the first test after the lessons on cylinders, the experimental group did significantly better ($p < .000$) with a small effect ($d = 0.48$). The comparing test after the lessons is not normally distributed; therefore, the Mann-Whitney test was used. Regarding the follow-up test, only a few classes agreed to write the test after half a year. For these classes, we used the paired t-test to see if there is a change in the individual results after half a year without any lesson about cylinders. The experimental group scored nearly the same as before, giving us a p-Value of .384 and an effect size of $d = 0.034$. However, the control group did score worse than before; the p-Value is .000 and the effect size $d = -1.15$ (see Table 2 for details).

Table 2. Differences between the First Test and the Follow up Test (German Study)

Group	*N*	*M*	*SD*	*P*	*D*
Control	37	-1.838	1.860	0.000	-1.150
Experimental	42	0.071	1.538	0.384	0.034

For a more detailed view of the learning process, the smartphones' log files (see Figure 5) are analyzed. As mentioned above, we cannot only divide into control and experimental group. The data reveals who did successfully solve a task in the experimental group. Since the cylinder tasks on the trail correspond to typical textbook tasks, we can compare the success rate of the comparing test for pupils in the control group and the experimental group and have solved no corresponding task, one corresponding task or two corresponding tasks. The cross-table shows that those pupils who have solved a maths trail task performed better on a similar task in the comparison test than the pupils from the control group and the pupils from the experimental group who have not solved such a task during the maths trail. The effect of the treatment can be traced down to the single maths trail tasks (see Table 3 for an example). The odd ratios are between 2 and 3 for nearly every maths trail task (one has an outstanding odds ratio of 6). It was two or three times more likely to solve a textbook task for a pupil who has met a similar maths trail task than a pupil who has only worked with textbook tasks before.

Table 3. Effects on Solving Maths Trail Tasks to Solving Textbook Task

		Related tasks in the treatment		
	Control	*None solved*	*One solved*	*Both solved*
Textbook task solved	10%	7%	23%	33%
Textbook task not solved	90%	93%	77%	67%

Regarding the IMI survey: with mean values of 4.7 and 4.5 (scale: 1 – 7), the subscales (1) enjoyment and (3) usefulness are significantly higher than the theoretical average value of 4 ($p < .01$). The perceived competence (2) mean score of 3.9 can be considered average. The pressure subscale (4) mean value (2.7) is lower than average. The subscales indicate that the overall experience of walking a maths trail supported by a smartphone application was perceived positively regarding motivational aspects. Students enjoyed the activity and had the feeling that it was helpful to them. In contrast, Fredricks and Eccles (2002) found that students general interest in mathematics decreases over the school years. In grade nine, the mean value is 3.7 on a 7-point Likert scale.

Discussion

Walking a maths trail with the support of the MathCityMap application increases the pupils' performance and motivation. It has significant effects in the short- and long-term. However, it cannot be directly concluded that this all came from the mobile technology used. Both studies did not try to run a maths trail without technology to compare that to a mobile app supported maths trail. That is an idea for further research. Nevertheless, we think that we have valuable clues to state that mobile technology got its share in the increase in performance and motivation. First of all, mobile technology made it possible for a whole class to do a maths trail "on their own". Because the app gives hints and feedback, the teacher does not have to be around in most cases when the pupils solve a task. Furthermore, being on their own implied an individual autonomy for the pupils, known as one pillar of intrinsic motivation, following Ryan and Deci (2000).

The second pillar of intrinsic motivation is perceived competence. Taking a look into the log files, pupils could only solve 37% of all opened tasks on the first attempt, but they solved 78% of them in the end. The difference of 41% is quite significant. Without technology, this 41% would have never solved the task right. They would produce a solution, and later on, in class or so, they would get feedback. Mobile technology offers the chance to get feedback on the solution while being at the object. In the case of negative feedback, pupils can try to solve the task again, which is successful in 41% of all tasks. The feedback itself makes it possible to perceive competence. It doubles the success rate, which is again very important for the performance since we know about the significant effects of solving a maths trail task on the classroom performance.

In the introduction, we wrote about the symbiosis of analogue and digital learning. With the support of a mobile app, the maths trail did not lose anything. All positive aspects stay the same. It is still an outdoor activity; pupils are going out, moving around, being in groups, discuss, measure and calculate. Furthermore, now the technology does not replace that but enriches it with meaningful aspects like hints, feedback, an integrated map where the tasks can be found and communication with the teacher.

Both parts, the analogue and the digital one, contribute to the success in a way the other cannot. Going out, touching the objects, seeing them, measuring and counting are analogue experiences that can hardly be replaced entirely digital. On the other hand, communication, hints and automatic feedback can hardly be replaced by analogue technics. We have not yet touched the themes of authoring, collaboration and sharing amongst teachers, which also became a lot easier with digital technology.

Summary

In both countries, Indonesia and Germany, maths trail activities instead of regular lessons increase the learning outcome. It is essential to point out that these maths trails are not add-ons to the regular lesson, no additional training program, but instead of regular lessons. We do not know to what extent the increase is possible and when ceiling effects will appear. Both studies concluded that the experimental group's mathematical performance was better than the control group shortly after the treatment. Besides, in Germany, the study shows that this is a long-term effect. The results have shown that the control group has forgotten what they have learned in class after half a year while the experimental group had a stable memory of what they have learned.

Besides all the positive aspects of a maths trail that could explain the increased performance, like the movement, the outdoor experience and the inactive learning, the results reveal that it strongly depends on the tasks themselves if the performance increases. Only if the pupils solve a similar task in the maths trail solved the related task in the comparison test significantly more often, which is somewhat surprising. Staying inside the classroom and performing textbook tasks have less effect on solving such textbook tasks than on a maths trail with similar tasks. We have to recall that the pupils will probably be able to work on more tasks inside the classroom. The average was eight solved tasks for the four outdoor lessons, two tasks in 45 minutes. This phenomenon was also noticed before by Rikala and Kankaanranta (2014). The data confirm the teacher's impression; pupils did practice less outdoors but got better exams afterwards. We can conclude that the maths trail setting increases the learning potential of the tasks. However, we do not know what would have happened if we had a control group to solve the outdoor tasks on paper inside the classroom. We need further research to uncover the reasons for success regarding maths trail tasks.

Regarding motivation, in both countries, the motivation scores were high. In Germany, the Intrinsic Motivation Inventory was used and revealed a higher motivation score for the experimental group than expected from the average pupil at that age, reported by the literature. The Indonesian study used the Self-Determination Index. The values of the experimental group were significantly higher than those of the control group. The follow-up maths trail activity and the survey one year later show that increased motivation is also a long-term effect. All of these findings are in perfect alignment with the previous qualitative studies before.

Overall, we highly recommend going out more often during the lessons, for example, to run a maths trail. The pupils benefit in ways of health but also increased performance and motivation. Researchers all over the world have found positive effects. Teachers can create maths trails everywhere. The maths trails are not limited to a specific place or structure, nor are special tools needed. This learning environment can be used in every country to go out with the pupils (at least to the schoolyard). Pupils can run maths trails with pen and paper or with new technology. Since mobile devices became common, it is possible to provide mobile supported maths trails for pupils. The technology enriches the maths trails with an exciting and motivating component.

Further research is needed on the long-term effects, both in motivation and performance. Generally, more research on performance is required to spot the reasons for a maths trail's success. Moreover, we are entirely missing research concerning effects on fields like spatial imagination, for example. Mathematics trails are an exciting learning environment that should be enriched by digital technology. A field we just have begun to touch yet.

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